Brief Descriptions of Some Attractive Voting Methods (2019)

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If one provides ordinal ballots for a collection of candidates where the voters are not allowed to truncate their ballots or be indifferent between two or more candidates, what are some of the election decision methods for selecting a single winner that are attractive? Here attractive will mean that they are easy to explain and/or have nice properties.

Figure 1 shows a typical ordinal or preference ballot, in this example for candidates 5 candidates, and indicating that 5 people voted for this ballot.

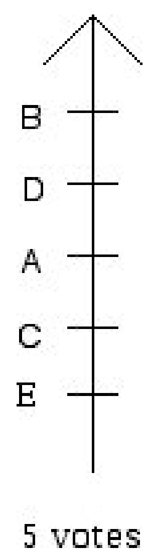

Figure 1

Thus, an election where there are n voters involves each voter producing a ballot which lists all of the alternative choices (candidates). In general, in deciding a winner we would have to specify what happens when ties occur. We will assume that some procedure exists for breaking ties or that examples have been chosen where ties don't occur.

In a particular election the number of first place-votes that a candidate Z receives is the number of voters who rank Z highest (first) on their ballots. The number of last-place votes that Z gets would be the number of ballots which rank Z last on their ballot. Thus, in Figure 1 E got 5 last-place votes.

A candidate has a majority if the number of first-place votes the candidate receives is more than 1/2 of the total number of votes cast.

Some election methods assign points to a candidate based on where on the ballot the candidate is positioned. The Borda Count is such a system and there are different approaches to how to assign the points. The approach I like is where, given a candidate X on a ballot, one counts the number of candidates below X and gives X this number of points. Thus, in Figure 1 for each voter with this ballot, B would get 4 points, D would get 3 points, A would get 2 points, C would get 1 point and E would get 0 points. The appealing thing about this approach is that it works even when ties on a ballot are allowed, something we are not allowing for the discussion here.

Figure 2 shows a typical election, no ties on the ballot of any individual and where there are 61 voters.

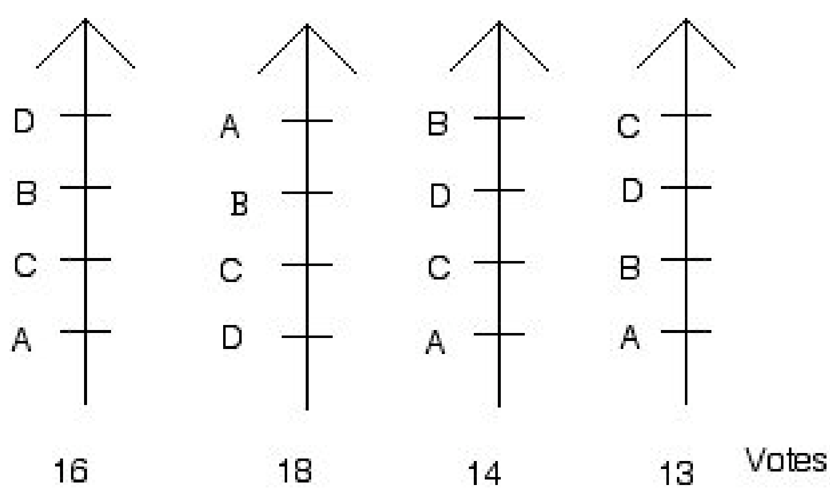

Figure 2

Note that since we have rankings for all of the candidates by every voter, we can carry out calculations for "run-off" elections that don't require voters to go to the polls another time. For example, in a two-way election involving only candidates D and A we would conclude that D gets 16 plus 14 plus 13 votes to A's 18 votes and that in such a two-way race D would win.

Methods

1. Plurality

The winner is that candidate who gets the largest number of first-place votes.

2. Ordinary run-off.

If no candidate gets a majority, then select the two candidates who got the largest number of first-place votes. Now hold an election between these two candidates, and whoever is the winner of this "two-way" race wins the election.

3. Sequential run-off (Instant run-off (IRV))

If no candidate has a majority, eliminate the candidate who has the fewest first-place votes. Repeat this procedure until one candidate is victorious.

4. Borda Count

For a candidate X and a given ballot b, assign X i points based on the fact that i other candidates are below X on ballot b and weight this number i by the number of voters who voted for a ballot identical to b.

5. Condorcet's Method

The winner of the election is that candidate Z who can beat every other candidate in a two-way race, assuming such a candidate exists.

Comment: There are elections where no Condorcet winner exists!

6. Nanson's Method (Nanson was an Australian mathematician.)

Named for Edward J. Nanson, the method first computes the Borda count for each candidate. All candidates whose Borda count is equal to or below the mean of the Borda count scores are eliminated. The ballots are now updated to eliminate the candidates just mentioned and the procedure (using the "new" Borda counts") repeated until a single winner remains.

7. Baldwin's Method (named for Joseph Baldwin, an Australian astronomer.)

The candidates are ranked on the basis of their Borda count totals. The candidate with the lowest Borda count is eliminated and the ballots "updated" to show that this candidate is no longer available to vote for. The procedure is repeated until a single winner is obtained.

8. Coombs' Method

If no candidate gets a majority, the candidate with the largest number of last-place votes is eliminated. The procedure is repeated until a single candidate emerges as the victor.

9. Bucklin's Method

If no candidate has a majority, then one adds the number of first- and second- place votes of the candidates on all of the ballots. If some candidate has a majority (or more) the candidate with the largest majority wins; otherwise, one adds third-place votes, etc. until a person with a largest majority emerges. (By majority is meant at least one half the total number of votes cast rounded up to the next integer.)

10 Minimax or Simpson's Method

Compute for each candidate the results of the two-way races and record the maximum value which a particular candidate lost by in a two-way race. The candidate whose maximum value (as above) is as small as possible is the winner of the election. If a candidate never loses in a two-way race, that candidate is the Condorcet winner and is the Minimax winner.

Comments:

1. Many of the methods above, though designed to produce a single winner, can be modified to construct a ranking of all the candidates based on the ballots cast (though ties might occur).

2. We tend to think of geometry as the "playground" where the adoption of alternative axioms defines things of interest. For example, one is interested in geometries where given a point P not on a line l there is exactly one line through P parallel to l (Euclidean geometry), there are no lines through P parallel to l (projective geometry), there are two or more lines through P parallel to l (Bolyai-Lobachevsky planes). However, axiomatics are of great interest in regard to election methods for selecting a single winner in an

election. There are various fairness axioms that one would like to see hold for an election method. Arrow's Theorem basically says that in elections with at least three alternatives, there are no methods which obey all of a small set of fairness requirements. However, one is still interested in exactly which nice properties each proposed "attractive" method from some point of view obeys. For example, monotonicity is the idea that more support can't hurt a candidate using this method. Plurality, an otherwise not attractive method, has this attractive property. However, sequential run-off has the property that giving a candidate more support may harm the candidate. (The reason is that more support may change the order of elimination of candidates and in the end have a negative effect on a particular candidate's being able to win.)

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