School Choice Activity (Summer 2019)

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While in the not too distant past students who wished to attend a public school for their K-12 education were required to attend a school which was near where they lived, a so-called neighborhood school, in recent years students and their parents are being offered a choice concerning what school students are able to attend. Without going into the full complexities of the issues here, the following activity is designed to help think through some of the "fairness" issues that come up in meeting the needs of schools to have students to teach and students to have schools they can attend. Realistic examples for New York City involve hundreds of schools and thousands of students; however, by using "toy" examples many of the issues that come up in how to implement school choice under realistic circumstances will emerge.

Imagine that we have a collection of students who will rank the schools they want to attend. Below, some examples have "indifference" allowed in the ratings, but in theoretical studies of problems of this sort, the mathematics is "more transparent" when ties are not allowed, and when schools are not allowed to refuse certain students and/or students would rather not have an assignment than be assigned to some schools. The students will specify their first choice, second choice, etc. up to the number of schools that are available to attend. (Realistic versions may limit students to a maximum, say, of 10 choices.) And the schools have "priorities" for the students. For example, schools may offer priority to students who have siblings who already attend the school. Some of the schools will be able to only accept a single student but some schools will be able to accept a "quota" of students. Student applicants know the number of slots available at a school when they apply.

In describing student preferences and school priorities X > Y for schools will mean that the school gives priority to student X over Y, while x = Y will mean that the school is indifferent between X and Y. Similarly for a student U > V means this student prefers school U over school V while U = V will mean that the student is indifferent between getting into school U or V. The notation U = V = W = Y will mean that the student is indifferent to all of the schools in this collection, U, V, W and Y. To indicate school 3 I will write s3 and to indicate student 4, I will write i4 (individual 4).

These examples are collectively drawn from the papers referenced.

Example 1

Each of the 3 schools can accept 1 student.

School priorities for students

s1: i1 > i3 > i2

s2: i2 > i1 > i3

s3: i2 > i1 > i3

Student preferences for schools

i1: s2 > s1 > s3

i2: s1 > s2 > s3

i3: s1 > s2 > s3

Example 2

Schools s1 and s2 have quotas of 2 students each, while schools s3 and s4 have quotas of 3 students each. (Total capacity 8.)

School priorities for students

s1: i1 > i2 > i3 > i4 > i5 > i6 > i7 > i8

s2: i3 > i5 > i4 > i8 > i7 > i2 > i1 > i6

s3: i5 > i3 > i1 > i7 > i2 > i8 > i6 > i4

s4: i6 > i8 > i7 > i4 > i2 > i3 > i5 > i1

Student preferences for schools

i1: s2 > s1 > s3 > s4

i2: s1 > s2 > s3 > s4

i3: s3 > s2 > s1 > s4

i4: s3 > s4 > s1 > s2

i5: s1 > s3 > s4 > s2

i6: s4 > s1 > s2 > s3

i7: s1 > s2 > s3 > s4

i8: s1 > s2 > s4 > s3

Example 3

Each of the 3 schools can accept 1 student.

School priorities for students

s1: i1 > i3 > i2

s2: i2 > i1 > i3

s3: i2 > i1 > i3

Student preferences for schools

i1: s2 > s1 > s3

i2: s1 > s2 > s3

i3: s1 > s2 > s3

Example 4

Each of the 3 schools can accept 1 student.

School priorities for students

s1: i1 > i3 > i2

s2: i2 > i3 > i1

s3: i2 > i1 > i3

Student preferences preferences for schools

i1: s2 > s1 > s3

i2: s1 > s3 > s2

i3: s1 > s2 > s3

Example 5

Schools s1, s2, s3, s4 have quotas of 2, 2, 1 and 1, respectively. (Total capacity 6.)

School priorities for students

s1: i5 > i1 > i2 > i3 > i4 = i6

s2: i5 > i6 > i3 > i1 = i2 = i4

s3: i4 > i5 > i6 > i1 = i2 = i3

s4: i5 > i6 > i1 = i2 = i3 = i4

Student preferences for schools

i1: s1 > s2 = s3 = s4 = s5 = s6

i2: s1 > s2 = s3 = s4 = s5 = s6

i3: s1 > s2 > s3 > s4 = s5 = s6

i4: s3 > s1 = s2 > s4 = s5 = s6

i5: s3 > s1 > s2 > s4 > s5 = s6

i6: s3 > s1 > s2 > s4 > s5 = s6

Example 6

Schools s1, s2, s3 and have quotas (capacities) of 1, 2, and 1, respectively. (Total capacity 4..)

School priorities for students

s1: i2 > i4 > i1 > i3

s2: i1 > i2 > i4 > i3

s3: i1 > i3 > i4 > i2

Student preferences for schools:

i1: s1 > s3 > s2

i2: s2 > s1 > s3

i3: s3 > s2 > s1

i4: s3 > s1 > s2

References

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