Mathematical Modeling (Urban Operations Research) (Summer 2019) (Part 2)

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The activities in Part 1 can be treated at a variety of grade levels. Solving the problems that arise from these situations involves making assumptions about the information that has been provided. The purposes of the activities include:

a. Using a graph to help construct a mathematical model of information that comes up in everyday life.

b. Using a matrix to help construct a mathematical model of information that comes up in everyday life. (The matrix can be used to "store" the information which gives the taxicab or the Euclidean distance between the lettered points in the diagram.)

c. The difference between taxicab distance and Euclidean distance.

d. Situation 0 leads to the important mathematical model known as the Chinese Postman Problem. This problem generalizes ideas used by Leonard Euler in 1736 to solve the Königsberg Bridge problem, and which gave birth to a geometrical set of ideas known as graph theory. This problem involved the question of whether or not a collection of bridges joining various river banks and an island could be traversed once and only. Although this problem may not seem of much interest to urban students, when set as an operations research problem that involves municipal services it often catches their interest.

e. Situation 1 leads to the important mathematical model known as the traveling salesman problem (TSP). While this small problem can easily be solved by trial and error, large versions of the problem have been shown to belong to a class of computationally difficult problems. However, various easy "heuristics" (e.g. fast procedures that do not guarantee optimal answers) are often "rediscovered" by students. (Common names for these heuristics are "nearest neighbor" and "sorted edges.") When different distances/costs are used there can be different optimal routes.

f. Situation 2 leads to the important mathematical model known as the minimum cost spanning tree (MST) problem. This problem can be solved by trial and error but there are variants of the heuristics listed above that lead to "fast" algorithms for this problem. These algorithms are know as Kruskal's and Prim's methods. There is also a lovely "parallel" algorithm known as Boruvka's method which also will solve this problem.

g. Situation 3 leads to the important mathematical model known as the minimum weight matching problem. This problem can be solved by trial and error here but although it is known that there is a "fast" algorithm to solve the problem, it is rather involved. Special cases of this problem can be solved more easily and involve ideas about shortest path algorithms in graphs.

h. Situation 4 has connections to ideas in statistics. The ideas of range, midrange value, mean, median and mode can be discussed. It belongs to a class of problems known as facility location problems. One needs to decide what distance function is most appropriate and also which optimization objective should be used. What adjustment might make sense if there are different numbers of people located at A,...,F?

i. Situation 5 is related to the TSP (traveling salesman problem). It is often referred to as the TSP with neighborhoods. Instead of visiting particular points one must visit with a "simple tour" some point in each neighborhood.