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Committee on Mathematical Modeling Courses:

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The modeling process

Compared with established courses in areas of mathematics such as algebra, geometry, or number theory, mathematical modeling courses are of quite recent origin. Indeed, a standard definition of mathematical modeling does not yet exist. However, in general mathematical modeling is the use of mathematics to obtain insights into situations that arise outside of mathematics, such as physics, chemistry, biology, business and economics and daily life - in short, in all areas of knowledge. As jargon within mathematics, sometimes those situations external to mathematics per se are dubbed "real world situations."

Phenomena to model might include the trajectory of a firework (to ensure safety at a July 4th celebration), the scheduling of the operating rooms at a hospital (to serve more patients and cut down on waiting time by doctors and patients), the blending of different gasolines (to cut carbon emissions), the search for molecules that might make safe and effective drugs, the task of ranking pages based on a string entered into a Web browser, or the restoration of electricity after a hurricane, such as Hurricane Sandy that devastated Long Island, New Jersey and the New York City area.

Many applied mathematics courses do not treat modeling as a topic but concentrate entirely on mathematical concepts and techniques, especially differential equations, integral equations, transforms and other topics in analysis. These particular tools involve continuous mathematics but many of these tools have discrete analogs. New continuous and discrete mathematical tools are constantly being developed. Modeling may draw on a wide range of mathematical tools but, crucially, embeds the use of these tools in a larger framework, non-mathematical insights. Key aspects of the modeling process include:

* looking for new problems to analyze;

* recasting verbal or graphic descriptions of problems in mathematical terms;

* making simplifying assumptions which allows a problem with many complex aspects to be replaced by a more focused simpler problem amenable to mathematical analysis;

* developing new mathematical methods or modifying or combining old methods for attacking these problems;

* interpreting mathematical results in the context in which the problem arose.

* and, mathematical modeling goes beyond problem solving in that simplifying assumptions must be made, sometimes data collected, and after a solution is found, an attempt must be made to see if the results are insightful for the original situation.

The goal of this process is to achieve insights into the "real world" and, perhaps also, to further develop the "pure mathematics" tools that are fundamental to solving the problem.

While an area of mathematics like number theory might draw on analysis, combinatorics, geometry, and many other parts of mathematics to provide gain understanding into old and emerging problems of interest to number theorists, mathematical modeling not only draws on other areas within mathematics but also builds direct ties between mathematics and other disciplines. For example, research on DNA strings has motivated a variety of problems in the developing area of the combinatorics of words and pattern matching, but constructing mathematical models to find genes in the DNA of a recently sequenced species raises many complex questions. To solve them, one might have to know a significant amount of biology and study the biology of genes as well as the combinatorics of words.

Learning goals of modeling courses

Regardless of students' background and of the specific subjects covered, we argue that certain goals should be common to all modeling courses:

* Make students aware of the vast range of environments in which mathematics can play a vital role. In particular, students should see that

mathematics can be applied in the behavioral and social sciences and perhaps even in the humanities, as well as in the physical sciences and engineering.

* Develop students' ability to formulate mathematical problems and to choose appropriate techniques for their solution. We note that these skills are crucial in pure as well as in applied mathematics.

* Develop students' ability to communicate professionally to a variety of audiences, including the “end users” of a modeling problem, who may be utterly unfamiliar with the mathematics used.

Modeling and professional opportunities

The recently adopted Common Core State Standards - Mathematics has as its Mathematical Practice 4 - Model with Mathematics (full quotation in the Appendix). This provides a strong impetus for colleges to require or encourage future teachers to gain experience with modeling. In addition, the initial development of modeling courses partially coincided with the tremendous increase in numbers of students obtaining higher education that occurred in conjunction with the space race of the 1960's and the political events of the late 60's and 70's. Coursework in modeling helps students who finish their mathematical studies with the bachelor's degree to find jobs where they can put their mathematical skills to work. For these reasons, it is desirable not only to offer courses in modeling *per se*, but also to incorporate as much modeling as possible into other courses, especially those involving applied mathematics.

Modeling and applied mathematics in the curriculum

A strong distinction is often drawn between pure or theoretical mathematics and applied mathematics. College mathematics is most often taught by faculty who have advanced degrees in mathematics and whose background lies in the “pure” side of the discipline. After all, relatively few of the doctoral degrees granted in mathematics every year are classified under the title of "applied mathematics." However, most students who major in mathematics do not go on to pursue advanced degrees in theoretical mathematics. For this reason, we believe that undergraduate instructors should devote more attention to applications in most courses. This can be a challenge, either exciting or daunting, for faculty who see themselves as theoretical mathematicians, and who may themselves have had little exposure to applied mathematics.

Traditionally, mathematics majors have taken a core curriculum (Calculus, linear algebra, abstract algebra, etc.) with room for electives. This core has not necessarily included a course treating modeling or even substantial applications. However, in the last 10 years many mathematics departments have moved to degree programs in which students choose one of a variety of tracks. There may be a common core across tracks, but fundamentally each track has its own core courses and electives. When an applied mathematics track is an option, a modeling course may be offered. However, the non-applied tracks may not explicitly treat applications and modeling in a way that best serves students.

We suggest that a suitably designed modeling course cannot only strengthen the background of students already committed to mathematics, but also help recruit students to the mathematics major. Conversely, experiences in modeling can lead to deep questions about the mathematical tools being used and thereby help students to appreciate the need for rigor.

Modeling courses and student preparation

Modeling courses come in many flavors. This variety reflects, in part, the diverse situations that lend themselves to mathematical modeling. However, it also reflects the diverse backgrounds of students enrolled in such courses. If students already have experience with many areas of mathematics, they can bring more tools to bear on a modeling problem. However, if a course has a long list of prerequisites (calculus, linear algebra, discrete mathematics, differential equations, probability, statistics, etc.), then this limits the expected enrollment. This can be especially problematic at small schools or schools where few students major in mathematics. By contrast, a modeling course with relatively few prerequisites can help attract students to the mathematics major by enhancing their employability as well as their interest in the subject.

We also note that broad contrasts can be drawn between types of mathematical models, which may be continuous or discrete and deterministic or probabilistic. Some students may have a particular aptitude for working with one type of model above all. Touching on several different types of models might therefore give more students a chance to shine during the course of a semester or quarter.

Types of modeling courses

Regardless (almost) of the mathematical preparation of students, modeling courses may be divided into two broad groups. In one approach, the class begins with a verbal description of a real life problem, formulates models, and develops or chooses mathematical tools to attack the chosen problems for study. Thus, one might start with a problem about airline schedules that leads to a formulation as a linear or integer programming problem. In the other approach, the class centers on specific mathematical methodologies that students might not have seen already, with illustrations from real world situations where these tools might be employed. For example, one might have a unit on linear or integer programming concepts and show how these tools can be used to solve an airline scheduling problem. Or one might choose to show the power of Markov chains by studying them for their own sake and then showing how they can be valuable in a variety of applied situations involving genetics, marketing or sports.

In the Appendix we describe four different types of modeling courses. The diversity of both modeling problems and student audiences has led us to de-emphasize specific lists of topics for the courses, as might be appropriate for, say, linear algebra. Instead, we have tried to highlight the different niches that the four courses mentioned might fill at schools with different characteristics. What is appropriate for a school with 30,000 students and a full range of graduate programs might not work at a college with 600 students and no graduate courses. Likewise, picking the right approach for a particular institution involves understanding the career paths of its mathematics majors. Are they likely to go into teaching? Business and finance? Engineering, or even law? The answers must shape the content of the course.

Modeling and the computer

Students in a modeling course should learn by example and experience that the formulation and analysis of a variety of mathematical models can be carried out entirely “by hand” without the use of mathematical software. However, students should also learn how to judge when computers (including calculators) can be helpful or essential for modeling and how to choose appropriate software in this case. If mathematics majors are required to learn to use a specific software package, such as Matlab, Maple or Mathematica, in a class separate from the modeling course, then the instructor can assume a basic competence and build on students' knowledge by showing how the software can best be deployed to solve specific problems. If no such experience can be assumed, however, instructors should not underestimate the time and effort required for students to learn to use a sophisticated software package.

Mathematical modeling and statistics

The discipline of statistics offers data analysis techniques that frequently form a crucial bridge between raw data and mathematical models. It is therefore desirable to include in a modeling course at least one substantial problem or unit that requires students to use a statistical method (e.g. linear regression) in problem-solving.

Traditionally, mathematical modeling courses have not emphasized the role of using statistics for modeling purposes. Given the range of problems and tools of a non-statistical variety that can be used to get mathematical insight into a real world situation, it may be desirable not to greatly emphasize statistics in a modeling course. On the other hand, many mathematics majors are not required to learn very much statistics. These students may graduate with less knowledge of statistics than would be the case if they took a liberal arts course where statistics was a unit or took a service course offered by the mathematics department (which cannot usually be used for credit towards a mathematics major) with a title such as "Introduction to Statistics." Since new standards for K-12 mathematics instruction include probability and statistics, it is especially desirable to include a statistical unit in the modeling course if the audience includes future teachers who otherwise might not be exposed to statistical reasoning.

Assignments and assessments in modeling courses

More standard assignments and assessments:

* Drill problems to give students practice in applying new mathematical tools

* Problems in which students (individually or in groups) develop new models by modifying or extending models developed in class or in a textbook

* Problems in which students (individually or in groups) analyze parametrized models and report the results. For example, students might find the fraction of a population that must be vaccinated to prevent an epidemic (the critical vaccination threshold), given specific values of model parameters.

* Assignments in which students explore how predictions derived from a model depend on the model's parameters. For example, students might describe how the critical vaccination threshold depends on the mean number of secondary infections derived from one infected individual (the effective reproductive number) and the duration of immunity provided by the vaccine.

* In-class exams to assess mastery of the skills developed through the assignments described above.

Assignments and assessments of a type less familiar to many mathematics instructors:

* Instructors should assign essay questions throughout the term and provide guidelines for style and content of written work that correspond to a grading rubric. Mathematics instructors sometimes feel at sea when it comes to assessing technical prose, but guidance is available.

* An extended project culminating in an essay and/or presentation is desirable for two reasons. First, it requires students to integrate all the skills they are expected to acquire in the course. Second, it requires students to explain their work to an audience not intimately familiar with the problem. Indeed, it can be useful for students to write an “executive summary” of their report intended for an audience that is completely unfamiliar with the relevant mathematics. The nature of the project, and in particular the degree to which students are on their own in identifying a problem, creating a suitable model and choosing appropriate methods for its analysis, will depend on the students' background and maturity.

Reference:

Pollak, H.O., What is Mathematical Modeling?, in Mathematical Modeling Handbook, ed. H. Gould, D. Murray, A. Sanfratello, COMAP, Bedford, MA., 2011, pp. vii-xi.

Appendices:

Appendix I

Recommendations

Recommendations related to applied mathematics and mathematical modeling appear below. They are given in no particular order.

1. Calculus courses should specifically include modeling tasks which show how what one learns in Calculus lays the foundation for what was to become "classical applied mathematics."

2. Provisions should be made that all students who specifically plan to do K-12 teaching and major in mathematics be exposed to mathematical modeling, ideally in a course with this title.

3. Mathematical modeling and applications of mathematics should be infused in all courses, no matter how theoretically their material is viewed.

Comment: One way to accomplish this infusion is by using writing projects which require students to learn about applications related to the theory that they are covering in a course.

4. Students should be encouraged to take discrete mathematics early in their mathematical careers to broaden their exposure to modeling situations and new mathematical tools. Mathematics departments could consider offering parallel tracks to enter the mathematics major (e.g. both as requirements) via Calculus and discrete mathematics. Calculus and discrete mathematics courses offer many opportunities to sample mathematical modeling techniques.

5. Majors should be provided with career information and graduate program information that builds on using the applied mathematics and mathematical modeling skills they have acquired. Students with undergraduate degrees in mathematics often select to use their mathematical skills in the business world, so attention should be paid to relating how American businesses (large and small) rely on mathematics beyond arithmetic, algebra and generalized reasoning skills.

Appendix II. Four sample modeling courses

To give faculty maximal flexibility, rather than specify syllabi for three of the courses envisioned, a generic syllabus is offered. For the fourth course, a more prescriptive collection of topics is offered. For all four courses different suggestions are made, organized in three different ways: subject area matters, mathematical tools, and themes for giving the students a modeling/applications experience that is both broad and deep.

Learning objectives for all of these courses:

a. Expose students to a broad collection of areas to which mathematics can be applied.

b. Develop mathematical modeling and problem solving skills.

c. Learn new mathematical tools to solve problems that can arise outside or within mathematics.

Modeling Course I

Audience: Highly prepared students - meaning those who have completed a three-semester calculus sequence and linear algebra as well as advanced or elective courses such as differential equations, discrete mathematics, or probability.

This course would be appropriate for schools with sufficiently large programs that having a course with many prerequisites would not make it hard for the class to run. It could serve as a capstone course option for the mathematics major or applied mathematics tracks.

Modeling Course II

Audience: Mathematics majors who have not necessarily yet taken a large number of mathematics courses.

This course could serve as an introduction to applied mathematics for students who may plan further coursework in this area.

Modeling Course III

Audience: Mathematics majors, with a focus on the needs of those majors who hope to pursue a career in pre-college mathematics teaching.

This course is aimed at mathematics majors who plan to teach in middle school and high school. The purpose of the course is to make students familiar with models which relate to and are of value to the curriculum of the CCSS-M (Common Core State Standards - Mathematics).

Modeling Course IV

Audience: Prospective elementary school teachers, and liberal arts students in general.

This course would have no prerequisites beyond high school algebra. The course might be recommended or required by the college for students as part of a program to train elementary school teachers. Given the importance of statistical thinking in modern society, at least one unit in this course

should focus on statistics.
Generic course description (Courses I, II, III)

Ways of using mathematics in subjects outside of mathematics will be developed. Real world situations will be examined and mathematical tools (continuous, discrete) applied to them in order to get insight. Situations will be drawn from different areas of knowledge and new mathematical tools will be developed as needed to analyze the situations examined.

Units for Course IV (Modeling for elementary school teachers and liberal arts students)

a. Statistics and Probability
b. Daily life models (urban operations research, financial mathematics)
c. Elections and voting
d. Fairness models
e. Models in the physical sciences and social sciences

Need for background in trigonometry and algebra should be minimized and reviewed as necessary for Course IV.

What should actually be taught in each of the courses above? For courses with titles such as Modern Algebra, Linear Algebra, Number Theory, etc. there is a core of topics that typically are taught, perhaps augmented with special topics that reflect the interests of the instructor. By contrast, mathematical modeling courses as they exist today are remarkably diverse in both mathematical content and areas of application. Topics for modeling should in part be chosen with an eye to students' interests and career plans, as we have noted above. They can also be chosen to reflect current events: in the early 2010s, for example, “hot topics” might include climate change, networks (or electricity suppliers, social networks or inter-bank transactions), ad auctions, influenza epidemics, and many others. Finally, it is legitimate for each instructor's taste to inform the choice of topics. Indeed, faculty members unaccustomed to modeling may be more willing to teach the course if they have latitude in selecting areas of application. With this in mind, some different ways to come at modeling are offered, and, of course, instructors are free to use a mixture of approaches.

<u>Modeling approached through subject area</u>

What follows is a sample of subject areas where one can employ mathematical models. The illustrative topics are chosen with a "bias" towards topicality and historical importance. The lists here are purposely kept short!

Actuarial Science

Soundness of pension systems
Annuities
Pricing futures and options

Biology

Phylogenetic trees
Compartment models
Ecological models
Classical genetics (Hardy-Weinberg equation)

Business

Ad auctions
Mechanism design
Scheduling
Flexible manufacturing
Resource allocation

Chemistry

Dynamics of chemical reactions
Diffusion
Knotted molecules

Computer Science

Complexity and analysis of algorithms
Image analysis and manipulation
Random number generators
Codes and their complexity

Ecology

Climate change
Predator-prey
Animal and plant harvesting
Sustainability

Economics

Mechanism design
Market design
Nash equilibria
Game theory
Price of stability and price of anarchy

Engineering

Elasticity and rigidity
Signal processing
Acoustics
Control theory
Numerical methods

Genomics

Edit distance
Gene identification
Pattern matching
Phylogeny

Geology

Percolation through porous media

Medicine

Drug development
Medical imaging
Medical databases

Network science

Structure of large social networks

Operations research

Garbage collection, snow plowing, pot hole inspection, painting lane lines
School bus routing, meals on wheels, truck deliveries
Facility location

Physics

Ballistics
Planetary orbits and satellite orbits
Special relativity
Quantum mechanics

Psychology

Learning theory (work of William Estes)
Paired comparisons
Decision making and risk perception

<u>Modeling approaches in terms of mathematical issues/tools</u>

Continuous tools

a. Differential equations

* Newton's Law of Cooling
* Projective motion (falling bodies)
* Radioactive decay
* Population growth
    exponential growth
    logistic growth
* Animal harvesting
* Mixing problems
* Predator-prey

* Natural selection
* Diffusion between compartments

b. Derivatives

* Maximization
* Approximation of functions
* Stability analysis

c. Integrals

* Solving pure-time differential equations
* Solving autonomous equations with separation of variables

d. Partial differential equations

* Heat flow
* Diffusion through porous media

e. Fourier series

* Study of periodic phenomena
* Electrical engineering
* Quantum mechanics

f. Special functions and orthogonal polynomials

Discrete tools

a. Discrete-time dynamical systems (recursion equations)

* Population growth
* Animal harvesting Credit card and mortgage payments
* Annuities
* Newton's method
* Natural selection
* Diffusion in discrete time and space (mixing)

b. Graph theory models

* Garbage collection, snow removal, pot hole inspection (Chinese postman problem)
* School bus routing, meals on wheels, manufacture of integrated circuits (TSP)
* Critical path scheduling
* Six degrees of separation and connectivity of networks

c. Mathematical programming

* Resource allocation
* Scheduling
* Discrete optimization

Stochastic models

* Diffusion
* Birth/death processes
* Queuing theory

Queues at bridges, tunnels, supermarkets, banks, computer services

Modeling organized within a thematic framework

Below are some "themes" along which topics embedded in modeling courses can be examined. These lists are kept very brief and can be greatly expanded. The choices have been governed to show the huge variety of areas outside of mathematics where modeling can be brought into play. Examples have also been chosen with some "currency."

1. Optimization

* Maxima and minima problems (Calculus, calculus of variations, isoperimetry)
* Linear and integer programming (linear algebra)
* Urban operations research (graph theory, mathematical programming, queues)

2. Growth and Change

* Population growth for human, animals, bacterial and viral populations (differential equations and difference equations, exponential growth)
* Animal and plant harvesting
* Changes in temperature, rain patterns, availability of water, CO_2 in the atmosphere

3. Information

* Codes to hide information (security for ATM machines, phone calls of government officials, email security)
* Codes to compress information (cell phone, HDTV, DVD, CD compression systems)
* Codes to correct information (cell phone, HDTV, DVD, CD, error correction systems)
* Codes to synchronize information
* Bar codes and codes to track information

4. Fairness and Equity

* Elections
* Apportionment

* Bankruptcy
* Games
* Weighted voting
* Fair division
* School choice
* Kidney exchange

5. Risk

* Decision making with uncertainty
* Analysis of lotteries

6. Shape and Space

* The nature of physical space
* Relativity
* Image processing

7. Pattern and Symmetry

*Mathematics for design
*Mathematics in art

8. Order and Disorder

*Randomness
*Ramsey Theory

9. Reconstruction (from partial information)

*Samples to reconstruct information about a population
* Data mining

10. Conflict and Cooperation

* War and escalation of crises
* Congestion games
* Mechanisms for regulation of businesses

11. Similarity and Dissimilarity

* Antiterrorism

* Workplace security
* Image processing
* Face, fingerprint, and iris identification

12. Close Together and Far Apart

* Development of new drugs
* Distance between species, bird songs, viruses, etc. (Hamming and Levenschtein distance)

13. Unintuitive Behavior

* Prisoner's dilemma
* Braess's paradox
* Vaccination and HIV intervention programs
* Paradoxes in scheduling
* Voting paradoxes

Appendix III

Statement in the Common Core State Standards of Mathematical Practice 4 which deals with mathematical modeling.

Model with Mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Even for mathematics majors who are not planning to teach at the time they are students, making sure that modeling course students are aware of the importance of modeling and applications in the CCSS-M would be of value to our society in general.

Appendix IV

A non-exhaustive bibliography dealing with mathematical modeling.

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Appendix V

Mathematical modeling competitions

Modeling competitions have emerged for undergraduates and high school students.

Undergraduate competition developed by the Consortium for Mathematics and Its Applications

http://www.comap.com/undergraduate/contests/mcm/

A high school competition developed by the Consortium for Mathematics and its Applications

http://www.comap.com/high school/contests/himcm/about.html

A high school competition organized by SIAM

http://m3challenge.siam.org/

A general competition run in France by the Fédération Française des Jeux Mathématiques and the Société de Calcul Mathématique

http://scmsa.eu/archives/SCM_FFJM_Competitive_Game_2013_2014.pdf

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