The Calculus Escalator

Prepared by:

Joseph Malkevitch
Department of Mathematics
York College (CUNY)
Jamaica, New York 11451

email:

malkevitch@york.cuny.edu

web page:

http://york.cuny.edu/~malk

Most Americans see the importance of having widespread mathematics instruction as part of K - 12 education. However, the choice of the mathematics to teach in K -12 very much depends on what goals concern society. Societal goals in teaching mathematics typically include having an adequate supply of practitioners of STEM (Science, Technology, Engineering and Mathematics) subject careers, having citizens who can collect and work with data, having citizens aware of the uses that mathematics can be put to, and having all high school graduates be comfortable enough with "fundamental" mathematical skills so as to meet the demands of daily life and a wide range of future employers.

An escalator is a device that makes it possible to get from one place to another in a relatively fast way but once one gets on an escalator, it is hard to get off. The structure of the recent CCSS-M (Common Core State Standards in Mathematics) seems to me to be philosophically dominated by the fact that some people, especially in the mathematics research community (as well as many mathematics educators), see Calculus as the dominant entry point into careers in STEM subjects. Preparing students to take Calculus, thus, becomes more of a concern than it should. This concern with making sure that there are future practitioners, by people who probably in part became mathematicians because of their "native" mathematical ability, has resulted in an unwise choice for a curriculum which serves a much broader collection of students than those seeking STEM careers. Mathematics as a discipline is also poorly served by this approach, even though some feel that what is good for the mathematics community is of necessity good for society.

Those concerned with having students prepare for STEM major careers reason somewhat in the following manner. To be properly prepared for Calculus in college (or 12th grade in high school) one needs to have a significant knowledge of trigonometry. To do well at trig one needs, among other skills, to know the algebra of polynomials and rational functions and how to solve polynomial equations, as well as some aspects of Euclidean geometry. So, ..., in kindergarten students should be seeing ...... While this is a bit of a parody, I regret to say it seems to be the basic framework which some research mathematicians have brought to the design and structure of the CCSS-M. It is easy to think that what worked for one personally is good for everyone.

In oversimplified terms, prior to the recent adoption by many states of the CCSS-M, it was the NCTM (National Council of Teachers of Mathematics) "Standards" which were the basis for the design of K -12 mathematics education. The NCTM Standards (1989) were the result of a grass roots effort of the premier organization for K-12 teachers interested in mathematics. They were a response to a variety of forces which included:

* tracking of students by gender and race to the detriment of these groups and American society

* employing new technologies such as calculators and software as part of mathematics instruction (What sense did it make to flagellate students who had trouble learning the theory and practice of pencil and paper algorithms to divide or multiply 5-digit numbers when there were calculators that could do this easily? Furthermore, using these devices and software are the norm in the workplace.)

* encouraging the teaching of elementary topics which were outside the traditional curriculum of K-12, including some topics belonging to "discrete mathematics" (graph theory, combinatorics, matrices, difference equations, etc.) which were increasingly being used as the necessary topics for entry into computer science (more generally information science) and were of growing significance for STEM majors as well.

* teaching, using contexts, situations that come up in daily life and in future career paths, as a way to motivate the study of what might otherwise seem like very dry or technical topics, and simultaneously making students aware of the range of situations where mathematics was being used (e.g. mathematics had applications not only in the "hard sciences" but in business, biology, etc.)

Not only did the NCTM Standards urge a balance between understanding of mathematics and using motivational situations outside of mathematics to help with teaching the content of mathematics, they were also sensitive to issues about making students aware of the role mathematics has in their daily lives. Cell phones are certainly a gift from engineering and physics to mankind but without mathematics developed in the 20th century (error correction codes, data compression codes, and methods to assign frequencies to calls so that conflicts did not result) we would not have the cell phone technologies we do. High school graduates should be aware of the role that mathematics plays in getting insight into how to remove snow from streets efficiently, how to schedule operating rooms and nurses to run inpatient and outpatient operations at a hospital more efficiently, and how forecasting hurricanes is more accurate because of mathematics.

A child growing up in America learns that there are many tools that help one on a regular basis. How many people can first point to when they first learned how to use a screw driver, a hammer, or a saw? However, we all have used these basic tools and we have a sense of what professions use these tools on a more than casual basis. Yet, though mathematics has developed fundamental tools to get insights into a wide range of useful situations, tradition prevents these topics from becoming included as part of the K-12 mathematics curriculum. The new CCSS-M discourages the teaching of these new elementary tools because, in part, they interfere with the "Calculus escalator."

The NCTM Standards not only list some of these tools of elementary mathematics but explicitly talk about the teaching of mathematical modeling. The CCSS-M talk about being able to "model using mathematics" but don't talk much about mathematical modeling - and there are major differences between the points of view. One irony of the difference between the CCSS-M and NCTM's Standards is that the latter were written and developed by people who had much closer ties to practicing K-12 classroom teaching than those who developed the former. While the CCSS-M certainly had input from a wide spectrum of people, they were marshaled into existence in an amazingly short period of time, for something that has all the earmarks of a de facto "national curriculum" for mathematics by people with much less experience in K-12 curriculum design or teaching experience than previous standards efforts.

One reason that the design of a Calculus escalator is being defended is that the US must be competitive with other countries and presumably we don't compare well with other countries in international comparisons. I find it bizarre that the US would design a mathematics education curriculum that imitates that of countries which have characteristics so unlike those of the United States. We are country of over 300 million people with much more heterogeneity than many of the countries which "outshine" us in international testing. It is claimed that not only are the average US students outperformed by many other countries on these tests but also our best students. Yet, even correcting for differences in size, I don't see the "footprint" on mathematics of mathematicians trained K-doctorate in mathematics in Singapore.

Some parents have promoted the CCSS-M for what seems to me a peculiar reason - they are trying to optimize the chance of having their children get into an elite college. Students who do well in Calculus AP classes have something "hard" on their transcripts when they apply to an elite college (even if they have no interest in a STEM major). Ironically, many of these parents admit to being "mathaphobes", yet they would rather see their children exposed to a quite narrow curriculum in mathematics rather than a broad curriculum which shows more mathematical tools than the CCSS-M. Rather than thinking that we should have a curriculum in mathematics which is not a mile wide and an inch deep, it is precisely such a curriculum that I believe makes sense for America. It is sad that for the short run America has committed itself to such a narrow curriculum and one that wrongly buys into the Calculus escalator approach with its ties to the CCSS-M. America would be better served by having the NCTM 1989 Standards evolve further.

References

(Anonymous) Common Core State Standards in Mathematics, 2010.

Malkevitch, J., Discrete Mathematics and Public Perceptions of Mathematics, in Discrete Mathematics in the Schools, DIMACS Series in Discrete Mathematics and Theoretical Computer Science, vol. 36. Joseph G. Rosenstein, D. S. Franzblau, F. S. Roberts, Eds., American Mathematical Society, Providence, 1997, pp. 89-97.

NCTM, Curriculum and Evaluation Standards, Reston, Virginia, 1989.

NCTM, Principles and Standards for School Mathematics, Reston, Virginia, 2000.