Weighted Voting Games (2019)

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We will use the notation [Q; a, b, c, ..., z] to represent a weighted voting game with players 1, 2, 3, ..., where player 1 casts a votes, player 2, b votes, etc. In order for a "coalition" of players to act, the number of votes (weight) of the coalition must sum to Q or more.

A coalition whose weight is Q or more is called *winning*. A coalition C is called *minimal winning* if it is winning but no subset of C is also winning. A *dummy* is a player with positive weight but who is not a member of any minimal winning coalition. A player v is a veto player if any winning coalition must contain player v. Intuitively, a veto player is one who can prevent action being taken unless that player "approves." A veto player is not always a dictator and there can be more than one veto player.

1. Given the voting game G = [5; 4, 3, 2] write down all the winning coalitions for G. Write down all the minimal winning coalitions for G. Which if any of the players in these games are "dummies?"

2. Given the voting game G = [6; 4, 3, 2] write down all the winning coalitions for G. Write down all the minimal winning coalitions for G. Which if any of the players in these games are "dummies?"

3. Given the voting game G = [7; 4, 3, 2] write down all the winning coalitions for G. Write down all the minimal winning coalitions for G. Which if any of the players in these games are "dummies?"

4. Given the voting game G = [8; 4, 3, 2] write down all the winning coalitions

for G. Write down all the minimal winning coalitions for G. Which if any of the players in these games are "dummies?"

5. Given the voting games below, write down the minimal winning coalitions. Which if any of the players in these games are "dummies?" Which players are veto players?

a. [12; 6, 4, 3, 1]

b. [13; 7, 5, 4, 2]

c. [10; 7, 5, 4, 2]

d. [13; 9, 5, 4, 4]

e. [13; 9, 5, 4, 3]

6. Compute the Shapley, Coleman, and Banzhaf power for the players in the games in Exercise 5.

7. Give some examples of real world situations where it seems natural to use weighted voting games.

8. As [Q; 8, 4, 2, 1] varies from 1 to 15 list the minimal winning coalitions in each of the games involved.

Note: Some power indices give more credit to a player who is a member of a small winning coalition than a large minimal winning coalition. It seems reasonable that coalitions which have fewer members are more likely to come into existence.

References:

Banzhaf, J., Weighted voting does not work: a mathematical analysis, Rutgers Law Review, 19 (1965) 317-343.

Banzhaf, J., One man, 3.312 votes: A mathematical analysis of the electoral college, Villanova Law Review, 13 (1968) 304-332.

Lucas, W., Measuring power in weighted voting games, Chapter 9, in Political and Related Models, S. Brams, W. Lucas, and P. Straffin, Jr. (eds.), Springer-Verlag, New York, 1983, p. 183-238.

Taylor, A. and W. Zwicker, Simple Games, Princeton U. Press, Princeton, 1999

Shapley:

3 players

1 2 3

1 3 2

2 1 3

2 3 1

3 1 2

3 2 1

3 players

1 2 3

1 3 2

2 1 3

2 3 1

3 1 2

3 2 1

4 players:

1 2 3 4

1 2 4 3

1 3 2 4

1 3 4 2

1 4 2 3

1 4 3 2

2 1 3 4

2 1 4 3

2 3 1 4

2 3 4 1

2 4 1 3

2 4 3 1

3 1 2 4

3 1 4 2

3 2 1 4

3 2 4 1

3 1 2 4

3 1 4 2

3 4 1 2

3 4 2 1

4 1 2 3

4 1 3 2

4 2 1 3

4 2 3 1

4 3 1 2

4 3 2 1