

Paired Comparison Activity (2019)

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Suppose your feelings about eating fruit are being explored. Which fruits do you like more or less than other fruits? This is your "task." Given a pair of fruits, you have to decide which of the two fruits you prefer to the other.

Example: !

Banana and Apple

If you prefer a banana to an apple, you write $B > A$ and if you prefer an apple to a banana, you write $A > B$. You are not allowed to throw up your hands and say you like them equally. You are not allowed to say "I have never heard of these fruits and I cannot compare them." You cannot say, "I prefer a granny smith apple to a green grape but I prefer a red grape to a granny smith apple." The rules of this "game" are that you must decide which you like better for each pair, hopefully, based on your true and honest feelings.

Probably at some time in your life you have had an apple and a banana and know based on experience your feelings about their relative merits but sometimes we are asked to make choices without a proper knowledge base. Also, there is the issue of the best banana you have ever tasted compared with the worst apple you have ever tasted.

Here are the fruits to consider:

Apricot
Blueberry
Cherry
Grape
Orange
Peach
Raspberry

Question 1:

To compare the fruits above in pairs, how many comparisons will you have to carry out?

Question 2:

Carry out the task of making all the required comparisons!

Question 3:

Based on the data you produced, which fruit is your most favorite?

Question 4:

Based on the data you produced, which fruit is your least favorite?

Question 5:

Use a directed graph (digraph) to represent the data you produced. (Draw a diagram of this digraph. What do the dots represent? What do the "directed line segments" represent?)

Question 6:

Is there a way to use the digraph you produced in Question 5 to determine your most favorite and least favorite fruits?

Question 7:

Based on the data you produced, can you produce a "ranking" of the seven fruits? That is, are you able to say which fruit you liked best, which you liked second best, ..., which fruit you liked least?

Question 8:

Give examples of "applied" situations where in essence the same task is being required but the ranking and paired comparisons involve something other than fruit.

Question 9:

The questions above are framed around the paired comparison of fruits. There are many other choices of things which might change your perceptions of paired comparison as an "effective" tool for obtaining preference information. For example, instead of fruits consider the following list of topics that are sometimes taught as topics in algebra/arithmetic/geometry to 8th grade students:

- 1 = solving linear equations
- 2 = fractions, ratio, and proportion
- 3 = slope of a line
- 4 = evaluate functions (e.g. $f(x) = -3x - 9$, compute $f(-4)$)
- 5 = solving two linear equations with two unknowns
- 6 = Pythagorean theorem
- 7 = solving quadratic equations

Repeat what you did in Questions 1 - 8 where this time you interpret Topic A > Topic B (in the list above) to mean that you feel it is more important for 8th grade students to master Topic A than Topic B with no allowance for being indifferent and where you are working with topics rather than fruits. You have to decide what the word "master" means for you in this context.

Question 10:

What does it mean for a student in a mathematics class to "master" a topic that is in the curriculum?