Exploring a Mathematical Model (Apportionment) (2019)

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Many attempts to use mathematical modeling involve the study of "fair division" questions. Intuitively, the idea is that one has some group of claimants who have different levels of "claims" on some item of value. The problem concerns how much each of the claimants should get of the valued item. For example, in the apportionment problem the claims are based on population (or party vote) and the valued item is a share of seats in the legislature that each state (or politcal party) gets. For the US House of Representatives, each state must get at least one seat. In a weighted voting problem each claimant wants some degree of "influence" in sharing the "prize" of getting some action taken. How should the weights be assigned to claimants so as to properly give each claimant their "proper" amount of influence? In a bankruptcy problem the issue is that there is not enough to pay all claims fully, so how should the claimants be treated to make what they are given fair?

First one formulates a mathematical model, such as an apportionment problem (e.g. based on populations how many seats (a non-negative integer) should each state get out of the number h of seats h, a positive integer). Then the investigation often proceeds to the analysis of what fairness properties different algorithms have when obtaining a solution of a particular problem instance. However, from the point of view of students another important skill is finding "similar" kinds of situations where the same mathematical model applies to a greater or lesser degree. Mathematics grows when variant models lead to new fairness ideas or algorithms for solving these variant problems.

Example:

A suburban school district owns 13 school buses. What is a fair way of assigning buses to the 4 sections of the district based on the number of students who must be transported to their schools in the district for the current school year?

Sample instance:

District A: 600 students; District B: 400 students; District C 300 students; District C 200 students.

Activity

Develop a variety of modeling situations that share similarities and differences with the "apportionment problem."

a. In what ways does a particular one of these situations resemble the usual apportionment problem setting?

b. In what ways does a particular one of these situations not resemble the usual "apportionment problem?"

c. How might one modify the mathematics used to "solve" the usual apportionment problem to deal with the differences of the situation you are now exploring?

Comment:

While the apportionment problem tends to emphasize the fairness of the solution found, in many problems which share the flavor of the apportionment problem there are other issues that come into play, notably optimization questions. Optimization questions involve finding the "best" (cheapest, shortest, fastest, etc.) way to deal with a situation. Thus, for school buses assigned to parts of town, there may be issues not only of fairness but also of getting the children to school in an efficient way.