

Activity Involving Non-Zero Sum Games (2019)

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We have seen that 2×2 matrix games (two players; each player has a choice of two actions each time the game is played) have a rich theory but one which helps give advice to the players that most players will find cogent. The situation changes radically when one looks at non-zero sum 2×2 matrix games. For each of the games below, take the opportunity to play them against a fellow student or acquaintance. Pay attention to whether some of the ideas we learned about zero-sum games can be applied here. For example, is it meaningful to adopt the idea of using dominating strategy analysis? Does it seem reasonable or useful to discuss how the game might be played before play actually begins, and if the game is played many times, might it make sense to take stock of what has happened in earlier play of the game before playing the game some more? All of the games below have "symmetry" but you might want to think about what further complexities emerge when the symmetries are broken. For zero-sum games there is a natural sense in which a matrix game has a "value." This "value" is a number which represents how their fortunes will prosper from either a single play of the game or in terms of "expectation" if they play the game many times. No such clarity exists for 2×2 non-zero-sum games. While the theory of zero-sum games having a value extends naturally to games with n actions by each player (that is $n \times n$ games), while for non-zero sum games things get "worse" quickly. When attempts are made to construct mathematical models of conflict situations in the "real world," the fact is that for simplicity reasons one limits the number of players one constructs the model for (for example, the public often is an indirect participant in many such games but its payoffs are rarely looked at).

Game 1

	Column I	Column II
Row 1	(1, 1)	(-10, 4)
Row 2	(4, -10)	(-6, -6)

Game 2

	Column I	Column II
Row 1	(1, 1)	(-40, 20)
Row 2	(20, -40)	(-11, -11)

Game 3

	Column I	Column II
Row 1	(5, 5)	(-2, 6)
Row 2	(6, -2)	(3, 3)

Game 4

	Column I	Column II
Row 1	(5, 5)	(-4, 20)
Row 2	(20, -4)	(-3, -3)

Game 5

	Column I	Column II
Row 1	(4, 4)	(1, 20)
Row 2	(20, 1)	(2, 2)