

A Sampler of Fairness and Equity Mathematical Modeling Situations

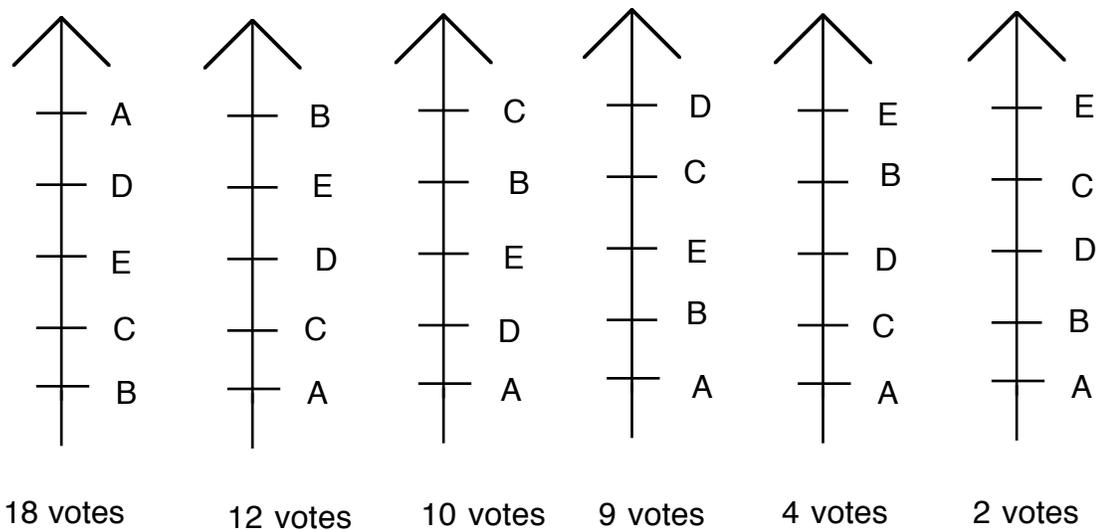
Joseph Malkevitch
Mathematics and Computing Dept.
York College (CUNY)
Jamaica, New York 11451

email: jmalkevitch@york.cuny.edu

web page: <http://york.cuny.edu/~malk>

Elections

The ballots below show the rankings of 55 Mathematics Club members for 5 choices of menus for the club picnic. Which menu deserves to be selected as the choice for the group?



(Note: There is no indifference; higher preferences towards the top.)

Bankruptcy

There are merchants (claimants) who have verified claims against the remaining assets of a small bankrupt company. What would be a fair way to settle the claims in each case?

Example 1:

Claimant	A	B	(Assets Remaining: \$120)
Claim	\$60	\$90	

Example 2:

Claimant	A	B	C	(Assets Remaining: \$200)
Claim	\$60	\$90	\$100	

Example 3:

Claimant	A	B	C	(Assets Remaining: \$200)
Claim	\$30	\$200	\$300	

Apportionment

The four divisions of a college, Arts, Sciences, Business, and Education have enrollments of:

2,180

1,880

1,420

640

A wealthy alumna of the college has decided to donate 30 scholarships to attract new students based on the different divisions' enrollment.

What is a fair way to assign the 30 scholarships to each division?

What would be a fair assignment if there were 31 scholarships?

Cost Sharing I

Towns A and B with populations of 12 and 8 (in hundreds of thousands) need to build a new water supply system.

Stand alone costs (in millions of dollars):

$$\{A\} = 160 \quad \{B\} = 90$$

$$\text{Joint cost: } \{A, B\} = 200$$

How should A and B proceed?

Cost Sharing II

Towns A, B, and C with populations of 12, 10, and 8 (in hundreds of thousands) need to build a new water supply system.

Example 1:

Stand alone costs (in millions of dollars):

$$\{A\} = 150 \quad \{B\} = 120 \quad \{C\} = 90$$

Joint costs:

$$\{A, B\} = 240 \quad \{A, C\} = 220 \quad \{B, C\} = 180$$

$$\{A, B, C\} = 325$$

How should A, B, and C proceed?

Example 2:

Stand alone costs (in millions of dollars):

$$\{A\} = 150 \quad \{B\} = 120 \quad \{C\} = 90$$

Joint costs:

$$\{A, B\} = 240 \quad \{A, C\} = 220 \quad \{B, C\} = 180$$

$$\{A, B, C\} = 315$$

How should A, B, and C proceed?

Legislative Fairness

The five towns in Rural County have populations of 7, 4, 3, 3, and 1 (in hundreds of thousands), respectively.

Is it fair for the representatives of these 5 towns to cast 7, 4, 3, 3, and 1 vote, respectively, where 10 votes are required to take action in the county legislature?

Games

Players named Row and Column independently choose a row (choice of two) and column (choice of two), respectively. The "payoff" is shown in the cell at the intersection of the chosen row/column pair. The payoffs shown are from Row's point of view. Column's payoff is the negative of the number shown.

	C ₁	C ₂
R ₁	5	-2
R ₂	-10	4

Is this a fair game?

How should Row and Column play the game to achieve his/her best result if the game is played:

a. Once?

b. Many times?

Transplants

Which of these factors should be taken into account in deciding whether or not someone is a suitable candidate for a transplant?

How famous the patient is?

How wealthy the patient is?

How old the patient is?

How sick the patient is?

How long the patient has been waiting for a transplant?

Things to consider:

1. What principles of fairness and equity do you think are essential in dealing with the situations above?
2. If everyone is "happy" with the way the situation is resolved, does this mean the solution method is a "good" one?
3. In some of the situations above the participants had some freedom to "lie" about some of the information provided. What are the consequences of this possibility?
4. What issues arise if, after a solution is effected, additional information becomes available (e.g. in the bankruptcy situation perhaps: a. another creditor appears; b. more assets are available than originally thought)?
5. Can you think of situations where being fair is not the only concern in attempting to deal with these situations?

Equity and Fairness Questions and Projects

1. How would one measure whether the distribution of income (or wealth) is more equitable in one country than in another?
2. How can one compare the fairness (equitability) of different tax systems?
3. In English, the words "fair" and "equitable" have somewhat different connotations. What are their differences? Can a procedure be "fair" but not "equitable?" Can a procedure be "equitable" but not "fair?"
4. For different areas of knowledge (e.g. accounting, medicine, business) can you find examples of situations where being fair and equitable are matters of concern?
5. Discuss how one can design fair election districts.

Short list of important references:

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