Example of Adams, Jefferson, and Webster Apportionment (2018)

Prepared by:

Joseph Malkevitch
Mathematics Department
York College (CUNY)
Jamaica, NY 11451

Y 11451

email:

malkevitch@york.cuny.edu

web page:

http://york.cuny.edu

Imagine that a high school senate has h = 25 members. How many seats should each of the classes below be assigned based on their class size?

The size of the Freshman (A), Sophomore (B), Junior (C), and Senior (D) classes are shown below:

A = 322; B = 256; C = 221; D = 188.

The total number of students is equal to 987. Since 987/25 = 39.48, each 39.48 students should be represented by one person in the college Senate. The number 39.48 is known as the ideal district size.

If we divide each of A, B, C, and D by 39.48 we get the exact quota for the number of seats each class is entitled to:

A: 322/39.48 = 8.156028369

B: 256/39.48 = 6.484295846

C: 221/39.48 = 5.597771023

D: 188/39.48 = 4.761904762

If we sum these numbers, but for any roundoff error we would get 25.

Webster's method rounds these numbers using the usual "rounding rule" that a fractional part more than .5 is rounded up and a fractional part with less than .5 is rounded down.

Using this rule: A gets 8 seats, B gets 6 seats, C gets 6 seats, and D gets 5 seats. Since these numbers add to 25 we are finished. We have apportioned exactly 25 seats. However, sometimes fewer than the required h seats to be apportioned are assigned and sometime more than the required h seats to be apportioned are assigned. For this example, had fewer than 25 seats been assigned we would have divided A, B, C, and D by a number smaller than 39.48 (this would make the quotients larger) so that after rounding (in the usual way) the number of seats assigned would have been 25. For this example, had larger than 25 seats been assigned we would have divided A, B, C, and D by a number larger than 39.48 (this would make the quotients smaller) so that after rounding (in the usual way) the number of seats assigned would have been 25.

Other methods of apportionment can be constructed by using a rounding rule different from the usual one. Jefferson's method is based on always rounding down, while Adams' method is based on always rounding up.

Suppose we were applying Jefferson's method to the example above.

Rounding the numbers above down would have resulted in A getting 8, B getting 6, C getting 5 and D getting 4 seats, which is a total of 23, which is 2 seats short.

Below is shown the result do dividing A, B, C and D by 35.

A: 322/35 = 9.2

B: 256/35 = 7.314285714

C: 221/35 = 6.314285714

D: 188/35 = 5.371428571

Rounding down we assign A nine seats, B seven seats, C six seats, and D five seats. Now 27 seats have been assigned so this is too many seats. Thus, dividing by 35 involved division by too small a value. Suppose next we try dividing by 37.

A: 322/37 = 8.702702703

B: 256/37 = 6.918918919

C: 221/37 = 5.972972973

D: 188/37 = 5.081081081

Again rounding down we give A eight seats, B six seats, C five seats, and D five seats. This assigns 24 seats, which is one less than what we have to assign. So let us divide by 36.6 this time.

A: 322/36.6 = 8.797814208

B: 256/36.6 = 6.994535519

C: 221/36.6 = 6.038251366

D: 188/36.6 = 5.136612022

Again rounding down we get A gets 8 seats, B gets 6 seats, C gets 6 seats, and D gets 5 seats. Note that typically there is a "range" of values that one could divide by to obtain the same apportionment of 25 seats. Thus, we have given away exactly 25 seats as required. This apportionment is known as the Jefferson apportionment, and in this example the results are the same as what we got by using Webster's method.

Adams' method works by always rounding up. Starting with the original calculation, repeated below:

A: 322/39.48 = 8.156028369

B: 256/39.48 = 6.484295846

C: 221/39.48 = 5.597771023

D: 188/39.48 = 4.761904762

we would assign A nine seats, B seven seats, C six seats and D five seats. This being 27, we have assigned to seats too many. Thus, instead of dividing by 39.48 we divide by value larger than 39.48, so that after rounding up we assign only 25 seats. Suppose we try using 41.

A: 322/41 = 7.853658537

B: 256/41 = 6.243902439

C: 221/41 = 5.390243902

D: 188/41 = 4.585365854

which rounding up assigns A eight seats, B seven seats, C six seats and D five seats. These numbers are still too large since we have assigned 26 seats. Next let us try dividing by 43.

A: 322/43 = 7.488372093

B: 256/43 = 5.953488372

C: 221/43 = 5.139534884

D: 188/43 = 4.372093023

This assigns 8 seats to A, 6 seats to B, 6 seats to C and 5 seats to D. This totals to 25.

Perhaps this work seems like a lot of tempest in a teapot because we used three different methods and got IDENTICAL apportionments.

Depending on our point of view things turn out not to be this lucky or simple. In general, these three different methods can yield three different answers starting with the same population (party vote data).

Practice the three algorithms above on this different data set, again with house size 25.

A: 916

B: 332

C: 244

D: 201

Do the three methods still give the same answers?

E.V. Huntington (Harvard) showed that "divisor methods" could be carried out using a "table" method which eliminates the "trial and error" aspects of what was done above, but is perhaps more "mysterious." Perhaps surprisingly, the mechanics of these different approaches only involves work with arithmetic.