DIMACS Series in Discrete Mathematics
and Theoretical Computer Science
Volume 36, 1997

Discrete Mathematics and Public Perceptions of Mathematics

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1. Introduction

A few years ago the Mattel Corporation marketed a talking Barbie doll, one of whose messages was "Math class is tough." Although this message from a talking doll was correctly greeted with great outrage by various sectors of the mathematics community because it conveyed a sexist message, perhaps most members of the general public would probably have agreed with Barbie. For these people, not only was math class tough, but mathematics itself was tough. Many people perceive mathematics to be tough because of what is commonly taught as mathematics in high school. Here is a list of the kinds of problems typically taught and tested for on standardized tests that attempt to measure success with high school mathematics:

Problem Set 1.

1. Factor:

$$x^3 + 5x^2 + 6x$$

2. Simplify:

$$(-2xy^2z^3)^3$$

3. Solve for x:

$$3(x-4) + 2(x-3) = x + 2$$

4. Add:

$$\frac{x+2}{x-6} + \frac{x-3}{x-4}$$

5. Find the value of the expression when $x = 3$ and $y = -2$:

$$(x^3)^2y - (xy)^2$$

1991 *Mathematics Subject Classification*. Primary 00A05, 00A35.

6. Prove that the lines through the vertices of a triangle that bisect its perimeter pass through a single point. Is the same statement true for the area bisectors that pass through the vertices of the triangle?

Compare this problem set with the following list of problems:

Problem Set 2.

A: Design an efficient route for a pot-hole inspection truck, which must inspect every stretch of street in the street network in Figure 1 at least once, and which starts and ends its tour at the location marked A. (You may assume that the streets are two-way.)

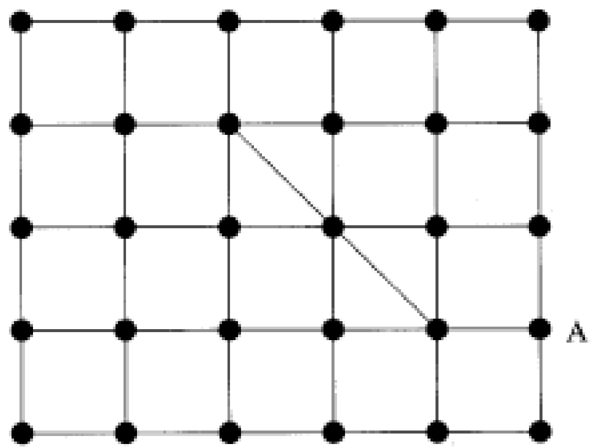

FIGURE 1

B: A small airport has three airlines that share the use of the runway at the airport. It has been decided that another runway must be constructed. What would be a fair system of allocating the cost of the new construction? (You may assume that it will be possible to obtain information such as number of flights per week, number of passengers served per week by these flights, as well as other passenger service and economic information concerning the three airlines.)

C: What would be a fair way for a divorcing couple to agree who should be given a book collection, a summer home, and some jewelry, other than selling the items and dividing the money equally?

D: A company wishes to create a decimal digit coding system for the products which it sells via a mail order catalogue. The code for each item is to consist of 9 information digits and a check digit. What are some of the considerations which might go into the design of the system?

E: The 55 ballots in Figure 2 have been collected for ranking 5 plays for a drama critics award. In this "preference schedule", the "18" at the bottom of the left column signifies that on 18 ballots the ranking of the five plays, from best to worst, was 1,4,5,3,2. Which play should be designated play of the year?

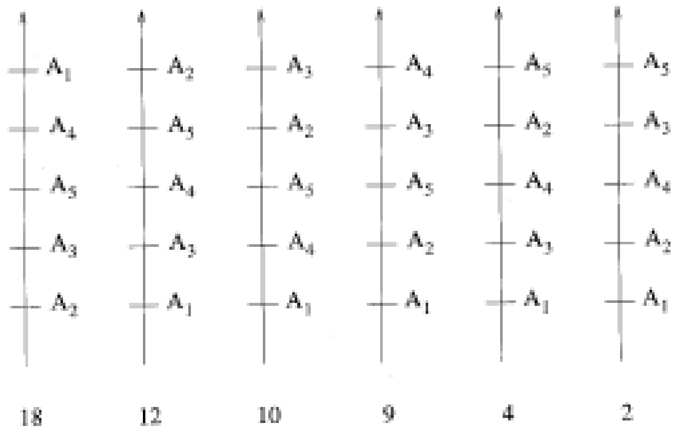

FIGURE 2

F: Figure 3 below shows the 12 courses that are being run by a small college during its summer session. An × means that the classes in that row and column have some students in common. If final examinations can be arranged in 4 time slots per day, is it possible to schedule all the final examinations in one day so that there is no conflict for any students among the times scheduled for the examinations for their courses?

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	–	×		×		×		×		×	×	×
2	×	–		×			×		×	×		
3			–		×	×	×				×	
4	×	×		–		×		×				×
5			×		–		×		×	×	×	
6	×		×	×		–		×				×
7		×	×		×		–		×	×		
8	×			×		×		–			×	
9		×			×		×		–	×		
10	×	×			×		×		×	–		
11	×		×		×			×			–	×
12	×			×		×					×	–

FIGURE 3

These two problem lists are worlds apart. The first list requires successful solvers of the problems to be comfortable with the manipulation of symbols, and each of the problems (other than the geometry problem) has (essentially) one correct answer. This list also gives no hint within the problems themselves of the ways in which mathematics influences daily life. By contrast, the second list (certainly in the statement of the problems) downplays the direct role symbols play, and the problems themselves point to areas of applicability. Perhaps the general public is justified in not seeing the total picture about mathematics, when overwhelmingly the exposure they have to mathematics consists of problems of the kind in Problem Set 1.

The current high school curriculum is the cause, in my opinion, of much of the negative image that the general public attaches to mathematics. The current curriculum is surprisingly often concerned with the kind of mathematics displayed in Problem Set 1. This is true despite the fact that this type of mathematics strays far from achieving many of the goals that the mathematics community and society in general hope can be achieved by teaching mathematics. These goals include the development of thinking skills, understanding of spatial concepts, and training for the workplace (see below for a larger list). In what follows I will try to explain what features of "discrete mathematics", and the way that it can be taught, make it a useful tool for changing the widespread negative perceptions about mathematics and for achieving society's goals for teaching mathematics.

2. What is Discrete Mathematics?

In this essay I am using the phrase discrete mathematics in a special way. Here, discrete mathematics will mean that collection of non-continuous mathematical ideas that have exploded in interest and study since World War II. In many cases these mathematical ideas had roots in much earlier times (e.g., graph theory was invented by Euler in 1736), but the invention of the digital computer served as a catalyst for the flowering of these ideas. Examples of mathematical tools falling within the rubric of discrete mathematics are: matrices, graphs and digraphs, difference equations, codes, and counting techniques. Areas of mathematics which fall primarily within the domain of discrete mathematics are ranking systems and social choice, graph theory, Markov chains, discrete optimization, combinatorics, and (discrete) probability. Just as for continuous mathematics, the study of discrete mathematics can be pursued for its own intellectual content or for specific applications. However, as we shall see, discrete mathematics lends itself to achieving some of the goals for mathematics education more effectively than what is currently taught.

3. Why study mathematics?

Mathematics differs from other areas of knowledge in that society has a vested interest in having the public have a breadth of mathematical skills. More than history or anthropology, for example, mathematics fulfills special needs of large sectors of American businesses as a knowledge base for their employees. Obviously, society has many interests to be served in promoting the teaching of mathematics. Here is a list of some of the many reasons offered for the importance and value of mathematics (in no particular order):

1. Promoting skills for enlightened citizens in a democracy.

2. Providing skills for workers in an increasingly technological society.
3. Providing understanding of the physical space in which we live.
4. Teaching logical thinking and analysis.
5. Serving as the language of science and engineering.
6. Encouraging flexible thinking when exposed to new situations.

What mathematics do we teach in high schools that is designed to convey to American these important aspects of mathematics? Currently, the content of high school mathematics can loosely be described as follows:

Grade 9: Algebra
Grade 10: Geometry
Grade 11: Algebra and Trigonometry
Grade 12: Precalculus; Calculus

In the context of this over-simplified account, you may wish to take a second look at the problems in set 1 above. The reason for this content in grades 9-12, while in many ways promoting the goals mentioned above, lies greatly in society's desire to allow students who are interested in pursuing careers in mathematics, computer science, science, and engineering to have the proper skills to begin college level work in these subjects. The entry course in college for the technologically-based professions, mathematics, and science is Calculus. Success in Calculus is tied to knowledge of algebra, trigonometry, and a subtle array of skills with functions and geometry. This fact, coupled with a tradition of teaching deductive geometry (transferred to America from England) and tradition in general, has given rise to the current curriculum. However, a few moments' thought, and a look at data concerning the portion of college graduates who pursue careers in science and mathematics, show that a high price is being paid for the current curriculum. Although the current curriculum is generally successful in locating the scientifically inclined, it results in vast numbers of other students who are "at sea" with the mathematics they are exposed to.

The bottom line for many students is that despite being exposed to mathematics continuously from Kindergarten through 10^{th} or 11^{th} grade, the typical high school graduate can not connect the value of the study of mathematics with what mathematicians really do. Put differently, students have learned when to "call" or hire a doctor, electrician, geologist, or plumber, but not when to "call" or hire a mathematician. For example, how many high school graduates know that mathematicians study optimization problems (i.e. finding the best or most efficient way of doing something) and fairness questions?

Another major failing of the current curriculum, from society's point of view, is that it does not show the dramatic way that mathematics has been involved in the development of new technologies. It is fair to say that without 20^{th} century mathematics it would have been impossible to accomplish the following dramatic achievements of science and engineering:

1. Landing a man on the moon.

2. Developing supersonic planes.
3. Developing more fuel-efficient cars.
4. Making CAT, PET, and MRI scans commonplace (i.e., breakthroughs in medical imaging).
5. Creating greater efficiency in American business operations (e.g., through the use of linear and integer programming models).

Although many people can in a general way see the connection between more fuel-efficient aircraft and mathematics, it would not be possible for these people to write down the mathematics involved, even in simplified terms. The reason for this is that many of the applications that people point to for demonstrating the importance of mathematics for technology involves the solution of differential and partial differential equations. This mathematics is not reasonably accessible for a high school graduate or even a college graduate (in areas outside of those with a scientific/mathematical focus). This contrasts sharply with the situation for discrete mathematics. Research problems in discrete mathematics are not likely to be resolved by typical high school students. However, for discrete mathematical problems, seeing the germ of the technical ideas of the mathematics and taking a few primitive steps with the mathematical ideas is possible with a much smaller knowledge base than would be the case for continuous mathematics. (For example, with no knowledge of algebra whatsoever one can go a long way in exploring graph theory and its applications.) Thus, altering the current curriculum to give a special role for ideas in discrete mathematics has much to recommend it.

Furthermore, many of the areas in which discrete mathematics is being applied, such as operations research, economics, and biology, are areas where average students have a richer background knowledge than for the fields where continuous mathematics is finding applications (i.e., physics, engineering, and chemistry).

4. Discrete Mathematics in our Schools

Mathematics should play an important role in our schools. Increasingly, knowledge of the role of mathematics in our technological society will be premium knowledge. This raises the issue about what concepts and ideas should be pursued as important ones in grades K-12 before differentiation of training occurs as part of career goals. My answer to this question is that we should make students aware that mathematics is involved with the following key areas and issues:

Optimization: What is the cheapest, fastest, best way of achieving a goal?

- What is the optimum blend of meats for a maker of cold cuts (i.e., salami or bologna) to put into the product, based on the costs of acquiring the meats that make up the mixture?

- What mixture of blends of gasoline should a company manufacture to optimize its profit?
- After a large storm, which forces the cancellation of many flights in a certain region of the country, what reshuffling of passengers, planes, and plane crews will restore the system to normalcy quickly and cheaply?

Fairness:

- How can one fairly divide an estate?
- How can one fairly divide property between a divorcing couple?
- What would constitute a fair way to fund schools?
- How can American election procedures be made more democratic?
- Is weighted voting a fair way to represent communities in a county legislature?
- What makes a game fair?
- What three-dimensional shapes are suitable for fair dice?
- How can two communities fairly divide the cost of constructing a water treatment plant that will benefit both communities?

Information:

- What codes would make it easy for businesses to transact their financial dealings cheaply, safely, and securely?
- How can errors in data transmission from outer space be corrected so that accurate images of planets and stellar objects are possible, even though the images are being sent with low power or unreliable transmitters?
- How can companies minimize the storage space they require for their records?
- Is it feasible to send high-definition television pictures along existing telephone wires?
- Can bar-code systems be designed that would speed the tracking of people or objects in a transportation system?

Risk:

- Is it safer to eat a vitamin that uses a non-natural color, to take a car ride, or to take an airplane ride?
- How likely is it that I will win a prize in a state lottery?
- How risky is it to gamble?
- What is the risk of using milk from cows that were fed genetically-engineered feeds?
- How dangerous are old nuclear power plants?

Growth and change:

- If current fishing patterns are continued, will the stock of a certain fish in the ocean be exhausted?
- What will the population of the world be in 50 years if current trends continue?

- How does an epidemic spread through a population?
- What pattern of market penetration should a company introducing a new product expect?
- How should a forest which contains trees that grow at different rates be managed?

Unintuitive behavior of complex systems:

- If weights in a voting game are proportional to population, is the power of the legislators proportional to the populations they represent?
- Can adding more processors to the scheduling of a collection of tasks increase the time to get the job done?
- If an additional road is built to relieve congestion, might congestion grow worse?
- Can one batter do better than another in each half of a baseball season, but do worse for the season overall?

Since discrete mathematics is a very broad area within mathematics, many more areas and application examples could be listed.

5. A Future Direction for Mathematics in our Schools

In light of the very negative view that people generally (and Barbie in particular) have of mathematics, it is highly desirable that actions be taken that would change these perceptions while at the same time providing students who are mathematically inclined with the stimulation that will allow that inclination to continue and flower. Discrete mathematics is a very fertile field to conduct experiments concerned with achieving this goal. Already at the college level, the so-called liberal arts mathematics course, historically taught with little regard to applications, has undergone a renaissance with the introduction of a new style of course based on an applied discrete mathematics curriculum (see [**1**]). There is thus reason to believe that an emphasis on discrete mathematics, delivered with teaching methods that keep the NCTM standards squarely in view, can transform the perception that mathematics students get in primary and secondary schools, while at the same time providing a steady stream of students to pursue careers in mathematics and science.

Acknowledgment

Many useful suggestions from the reviewers are gratefully appreciated.

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