The Diffraction Grating

1 Objectives

1. To understand interference in optical systems, and
2. To use a diffraction grating to measure spectral lines and laser emission wavelengths.

2 Introduction

In this lab, we're going to use macroscopic objects (discharge tubes, lasers, and meter sticks) to measure microscopic wavelengths via light transmission through a *diffraction grating*.

When you studied sinusoidal wave phenomena in PHYS151, you learned about phenomena such as *superposition* and *interference*: when waves from two or more sources propagate into a given volume, the individual waves retain their own properties and simply superpose, or sum. The resulting summed waveform will have areas where the individual waves tend to reinforce each other (constructive interference) and other areas will negate each other (destructive interference). In this lab, we will utilize gas discharge tubes and lasers in concert with a diffraction grating to provide a source of hundreds or thousands of waves, and use the resulting interference patterns to learn about the underlying light source.

3 Theory

Consider what happens if you have two coherent[1] light sources, S_1 and S_2; see Figure 1. Let's set them up so that the two sources are a distance d apart, and equidistant from a screen. Assume the distance from the screen to the sources, L, is much much larger than their separation ($d \gg L$). Wavefronts from the two waves will completely constructively interfere at a point on the screen when the light has to travel the same number of wavelengths from each source to that point; there will be a bright spot on the screen at that point. More generally, there will be a bright spots at every point on the screen where the travel distances differ by an integer multiple of the wavelength; similarly, complete destructive interference will occur if the travel distances differ by an odd half-multiple of the wavelength (0.5, 1.5, etc.).

Let's calculate where on the screen these bright spots will occur; we'll use the notation in Figure 1. We can define the location of the point P by the x and y offsets (L and y

[1] Two sources are coherent if they emit the same frequency of light with a vanishing relative phase angle.

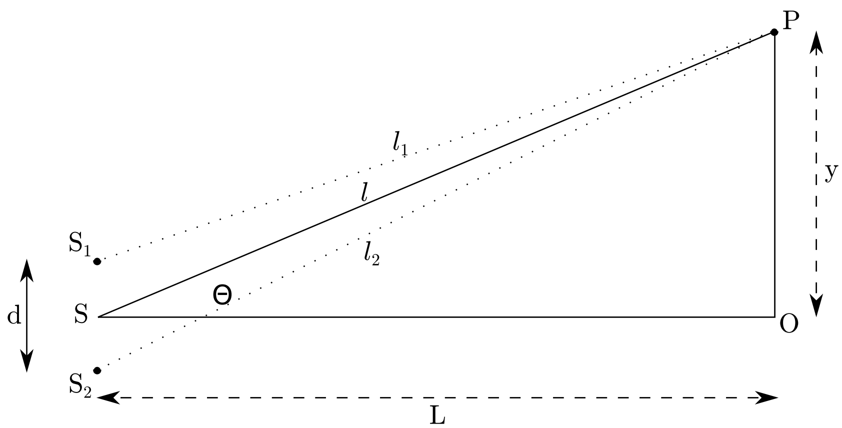

Figure 1: The relationship between the two light sources, S_1 and S_2, and the screen OP, along with definitions of the various distances quoted in the text.

respectively), as well as by the magnitude and angle (ℓ and θ). Of course, they are related by $\tan\theta = y/L$. Now, we know that the points of constructive interference occur when $\ell_2 - \ell_1 = n\lambda$, where n is an integer and λ is the wavelength of the sources. We can calculate ℓ_1 and ℓ_2 via the Pythagorean theorem

$$\ell_1^2 = \left(y - \frac{d}{2}\right)^2 + L^2 \qquad\qquad \ell_2^2 = \left(y + \frac{d}{2}\right)^2 + L^2 \ .$$

Now, by assumption $d \ll L$, and we'll also assume $d \ll y$. We can use Taylor's theorem, and expand each of the parenthesized factors

$$\left(y - \frac{d}{2}\right)^2 = y^2\left(1 - \frac{d}{2y}\right)^2 \approx y^2 - dy \ .$$

After expanding the similar terms in both of the ℓ_i, you should find

$$\ell_1^2\ell^2 + L^2 - dy \qquad\qquad \ell_2^2\ell^2 + L^2 + dy \ .$$

Taking the difference of these terms gives

$$\ell_2^2 - \ell_1^2 = (ell_2 - \ell_1)\,(ell_2 + \ell_1) \approx (ell_2 - \ell_1)\,2\ell = 2dy \ .$$

Divide both sides by 2ℓ, and substitute for the first term, and you should find for the maxima

$$n\lambda = \frac{dy}{\ell} = d\frac{y}{\sqrt{y^2 + L^2}} = d\sin\theta \ . \tag{1}$$

With only two coherent sources, the pattern on the screen will vary smoothly from one bright peak to the next dim valley to the next bright peak; in other words, we won't just see very bright peaks with no light in between them. If, however, we were to add another pair of sources, we would still get bright peaks according to Equation (1), but the spaces in between would be darker than in the two source case. If we keep adding sources until we

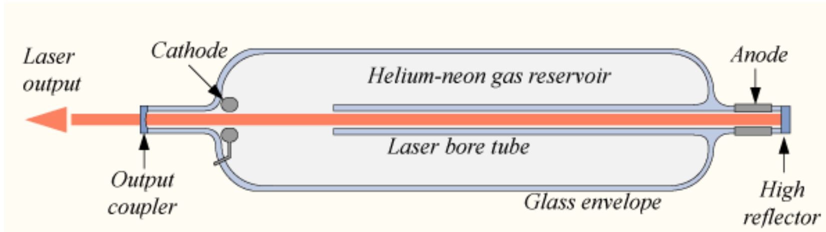

Figure 2: The construction of a Helium-Neon Laser consists of a gas discharge tube capped on each end with high-efficiency mirrors.

have a very large number of them, we would *still* find that we have bright peaks according to Equation (1), but essentially no light between those peaks; destructive interference between all the additional sources would essentially cancel all the light.

So, how do we obtain a large number of coherent sources? We don't. Instead, we obtain a *single* source, and we place an opaque layer of material between it and the screen. Then, we cut a series of very fine, closely and regularly spaced, parallel lines through the opaque material. At each slit, the light will emerge *as if* is was emerging from an independent source. The object we would have constructed is called a *diffraction grating*, and the spacing between the *line* of the grating is called the *grating element*, or line spacing, d.

4 Procedures

In the lab, there will be a number of gas discharge tubes, a Helium-Neon laser, diffraction gratings, measuring equipment, and various mounts and stands. First, you'll use a Helium-Neon laser, which operates at a well known wavelength, to determine the line spacing for your grating. Then, you'll use the grating to determine the emission wavelengths for a number of elements in gas discharge tubes.

4.1 Helium-Neon Laser

The helium-neon, or HeNe, laser was among the first continuous lasing systems ever developed. It's cheap, efficient, and produces its output in the optical part of the spectrum. This lasing system is essentially a gas discharge tube capped on each end with high efficiently mirrors; see Figure 2. The discharge excites Helium atoms to a set of long lived, metastable excited states; collisional interactions transfer this energy to the Neon atoms, exciting some of their electrons to the ${}^3s_2$ level. These transition to the ${}^2p_4$ state, emitting in the process a photon whose wavelength in air at STP is 632.816 nm.[2] This is right in the middle of the red band of the optical spectrum.

When we shine the laser through the grating, a single diffraction pattern will form on the screen. Since we can accurately measure the angular divergence of the first order peaks, we can accurately determine the grating element. That's your goal in the first part of the lab.

[2]The emitted wavelength is 632.991 nm in vacuum. Data gleaned from wikipedia.

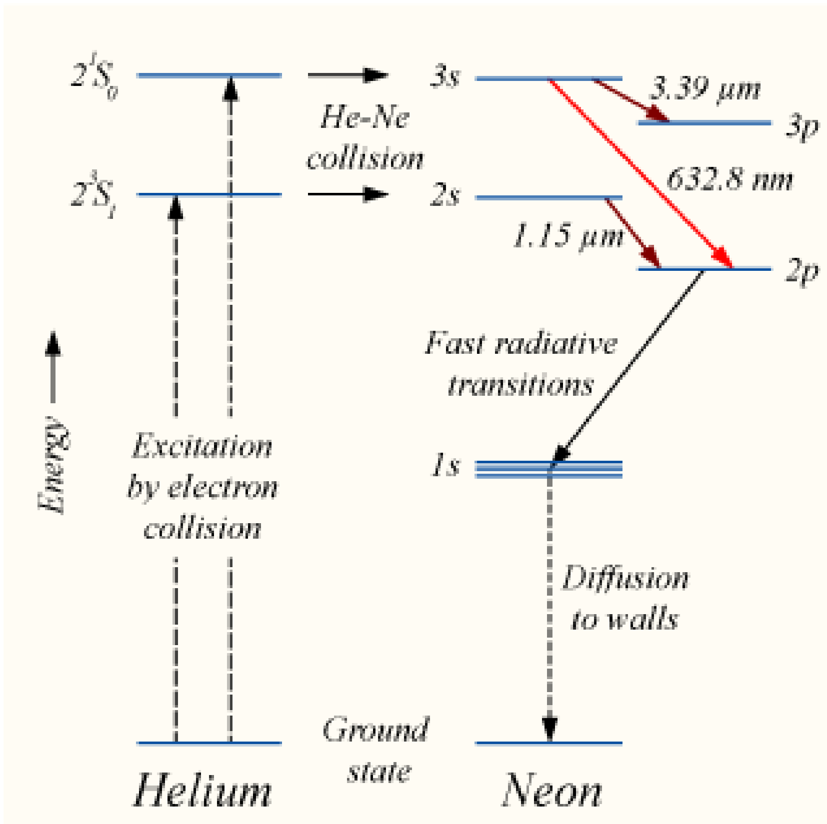

Figure 3: The level structure, and corresponding wavelengths, of the Helium-Neon laser system.

LASER SAFETY: Do not intentionally stare into the laser light! While a brief, inadvertent exposure to our HeNe lasers will not injure your eyes, intentionally staring into the beam for any length of time could give you anything from temporary vision impairment to permanent eye injuries, including blindness!

You are going to shine the laser on the wall, through the grating, and measure the positions of the zeroth and first order peaks. This data will give you the angle, and consequently, you can determine the grating element.

1. Mount the laser so that it shines on the wall or chalkboard. The beam should meet the board at a right angle.

2. Mount the grating parallel to the wall, so that the laser shines through it. Locate the first order peaks; the lights in the room will probably need to be off for this.

3. Carefully measure the distance from the grating to the wall (L), and the distance from the zeroth order peak to both first order peaks (y_1 and y_{-1}). The latter two should be the same; if they are not, either your beam is not normal to the wall, or your grating is not parallel to the wall.

4. Record these numbers carefully, and determine the grating element.

Now that you have the grating element, you can use your grating to determine the wavelengths of any number of light sources.

4.2 Laser Pointers

The red laser pointer uses a solid state laser diode to produce a beam in the optical spectrum. The exact wavelength is highly variable, and depends on the exact composition of the semiconductor diode, the coatings on the lenses in the optical system, and the temperature of the electronics. Still, they are *red*, and they are *lasers*, so the range of wavelengths output by a given pointer are very narrow. Determine the wavelength of your pointer.

1. Repeat the measurement of the previous section, but using a laser pointer instead of the HeNe laser unit.

2. Record L, y_1, and y_{-1}, and determine the wavelength the laser pointer.

4.3 Gas Discharge Tubes

In a gas discharge tube, a low pressure gas is ionized by a high voltage current flowing between two electrodes. The recombination of electrons onto the atoms results in the emission of specific wavelengths of electromagnetic radiation. Fluorescent lights are the most commonly encountered form of gas discharge tubes. They use mercury vapor, which emits a number of lines in the ultraviolet region of the spectrum to excite fluors coated on the inside surface of the glass, which then fluoresce across the optical band.

In contrast, we will study pure gasses in this lab, which will show us the behavior of specific atomic species. Some of these emission lines are in the optical region of the spectrum. Since these emission lines contain a vary narrow range of wavelengths, we can use a diffraction grating to determine the wavelengths of these lines. Different color light (different wavelengths) will be diffracted through different angles, much like in a prism. How does the ordering of colors compare with those in a prism?

> ***DISCHARGE TUBE SAFETY: These operate with a very high voltage between their terminals. Do NOT stick your fingers, pens, etc. into the terminals. Do NOT touch the ends of the tube when they are on. Do NOT remove the tubes when they are energized. Do NOT leave them on for more than a minute at a time. Also, the tubes get HOT when run for more than a few seconds; don't touch then until they have cooled down.***

Unlike the laser measurements made in the first two parts of the lab, the light from the discharge tubes is neither coherent, nor focused, and hence can't form an image on a screen or wall. Instead, you will be using your personal optical measurement system (ie, your retina) as the screen. This type of image is called a *virtual* image.

1. Fix a meter stick in the horizontal, and set a discharge tube vertically at the midpoint of the stick, and as close to the stick as possible. Mount the grating at a suitable distance (something like 50 cm should work) from the meter stick, with the grating lines parallel to the length of the discharge tube. Make sure grating is parallel to both the meter stick and the tube.

2. Look through the grating towards the discharge tube; keep your eye as close to the grating as possible to eliminate parallax shifts. Find the spectral lines, which will appear on both sides of the central maximum. If you did the previous step correctly, they should be symmetric about the central maximum. Adjust the distance so that the furthest first order line is as far out on the meter stick as possible.

3. Record the type of gas in the tube. Measure and record the distance from the discharge tube to the grating. Measure and record the colors and positions on the meter stick of all the first order lines on both sides of the central maximum.

4. Repeat with another gas.

Pre-Lab Exercises

Answer these questions as instructed on Blackboard; make sure to submit them before your lab session!

1. Why is the zeroth order maximum the same color as the source? For lines of a given order, are the red or violet lines on the screen closer to the zeroth order maximum?

2. What wavelength of light would produce a first order peak at 10°with a grating of 10^4 lines per centimeter?

3. A 340 nm wave with a grating of 5,000 lines per centimeter produces its first order peak at what angle?

4. Determine the grating element for a wavelength of 500 nm to produce a second order peak at 15°.

Post-Lab Exercises

1. From your HeNe data in Section 4.1, determine the grating element for your grating. Since both of your first order measurements should give the same result, combining them should give you a more accurate measurement. Estimate the uncertainty of your measurement.

2. From your data in Section 4.2, determine the wavelength of you laser pointer. Estimate the uncertainty of your measurement. Remember that the HeNe laser is also *red*, so the wavelengths won't be too far off. Don't forget to combine the data from both first order measurements.

3. For each of the gasses you measured in Section 4.3, determine all the first order line wavelengths. Estimate the uncertainties. Don't forget to combine the data from both first order measurements. These wavelengths are all well measured; look them up. How well do your measured values match the tabulated results?

4. Discuss briefly whether you have met the objectives of the lab exercises.