

# Kirchoff's Rules

## 1 Objectives

1. To verify Kirchoff's Loop rule for voltages, and
2. To verify Kirchoff's Junction rule for currents.

## 2 Introduction

Gustav Kirchoff was a prominent 19<sup>th</sup> century Prussian physicist. He is still well known for his work in both circuit theory and spectroscopy. He won every prominent award for science in the middle part of the century, and is remembered along with his spectroscopist colleague Robert Bunsen<sup>1</sup> by the Bunsen-Kirchoff Award for Spectroscopy.

The study of electricity was still in its infancy in the early half of the century; you may recall that Georg Ohm only discovered the relationship between voltage, current, and resistance in 1827. Kirchoff was certainly familiar with this work, and he first discovered what are now called Kirchoff's Circuit Laws during his electrical studies while a student in the 1840s. This later formed the basis for his doctoral dissertation.

## 3 Theory

Kirchoff's Rules easy to state, but first we need to define a few terms. A *loop* in a circuit is any closed path that travels through one or more of the circuit elements and returns to its origin. A *junction* is any point where two or more components are connected by a conducting path. With these definitions, the Rules themselves are simple to state:

1. Loop Rule: The directed sum of the voltage drops around any loop in a circuit must be zero:

$$\sum V_i = 0 .$$

2. Junction Rule: The directed sum of all currents entering a junction in any circuit must be zero:

$$\sum I_i = 0 .$$

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<sup>1</sup>Yes, *that* Bunsen.

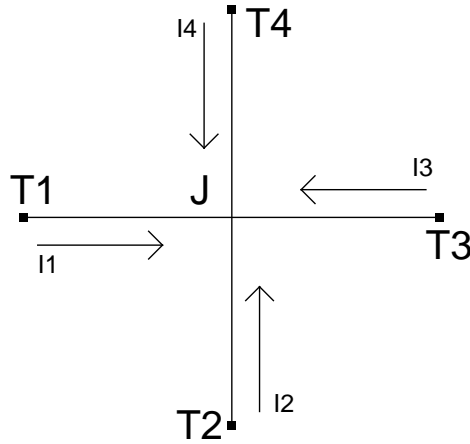


Figure 1: A junction,  $A$ , with four currents:  $I_1$ ,  $I_2$ ,  $I_3$ , and  $I_4$ .

The *voltage drop* is the difference in voltage from one terminal of a circuit element to another terminal. By using the term *directed sum*, we indicate that the *sign* of the term matters: if we choose to define a current entering a junction as positive, then one that leaves a junction must be negative; if we choose to define the current as traveling in a clockwise direction around a loop, then we should measure the voltage drops in the same direction.

Kirchoff's Rules are equivalent to the conservation of energy and the conservation of charge respectively. The junction rule is easy to understand in this context. Since current is just the motion of charged particles, the Junction Rule says simply that every charge that goes into a junction must leave it: you can't create or destroy charges. The Loop Rule is a little more complicated, but not much: because generating a voltage in a power supply requires a power source, and driving a current through a resistor dissipates power, all the energy that we add to the circuit must be consumed by the resistive elements.

Kirchoff's Rules apply to more than just resistors and power supplies (as we will learn in later labs), but they are really just approximations to the full theory of electromagnetism described by Maxwell's Equations. Luckily for you, these exceptions do not arise in any experiments we will perform in this class.

The *use* of these rules is relatively straightforward, but let's do some simple examples to make those uses concrete. Let's start with the Junction Rule. When we measure the currents *into* a junction, we mean that we *assume* that currents along all branches flow into the junction, and make our measurements assuming that's true. But since that can't be true - That's what the rule says, after all! - sometimes we'll measure the current along a branch as *negative*, which means that it actually flows *out* of the junction. Take Figure 1 as an example: there are four branches connected to junction  $J$ . Let's label the currents from points  $T_1$ ,  $T_2$ , etc by  $I_1$ ,  $I_2$ , etc, and let's further assume that they all flow *into*  $J$ . When we measure the current with our ammeter, we have to choose which terminal of the ammeter to place where. By the conventional choice of inflowing currents, we should *always* connect the negative terminal of the ammeter at  $J$ , while connecting the positive terminal to the  $T_i$ . Then, the meter will sometimes read positive values (for properly inflowing currents), and sometimes read negative values (for outflowing currents), and the sum of the total should

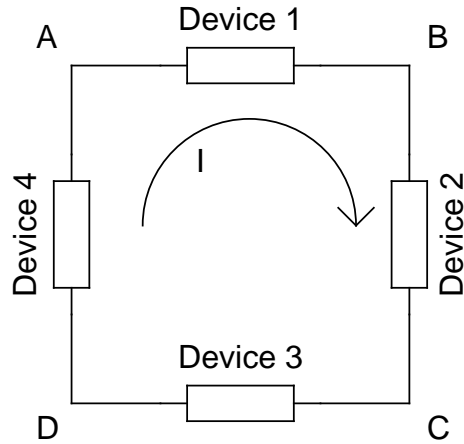


Figure 2: here be a caption

be zero. There are other possible approaches to the measurement, but all of them must be equivalent to what we have outlined here.

The approach for Loop Rule measurements is almost as simple: assume that there is a current flowing around the loop in one direction or the other, and measure all the voltage drops around the loop in that direction. So, for the loop in Figure 2, starting with node  $A$ , we'll assume a current flowing in a clockwise direction. First, measure the voltage drop between node  $A$  and node  $B$ , by measuring with the negative probe of the voltmeter at  $A$ , and the positive probe at  $B$ . Then, measure the voltage drop between nodes  $B$  and  $C$  by placing the negative probe at  $B$ , and the positive probe at  $C$ . Repeat all the way around the loop. Some of the voltage drops will be negative and some positive, while the accumulated voltage drop should vanish. As for the Loop Rule, there are other possible equivalent approaches to making this measurement, but they must all be equivalent to what we have outlined here.

## 4 Procedures

We will now investigate Kirchoff's Laws quantitatively, in two steps: first, we will use a single loop circuit to study the loop rule in isolation, and then we will add a second loop to study the loop and junction rules at the same time. For the experiment, you will need three multimeters, two power supplies, the circuit board, and various lengths of cabling. Use the two identical multimeters exclusively as ammeters, and the third multimeter for everything else.

### Reminder

When we say "Record a measurement", we *always* mean that you should record the measurement, *and an estimate of the uncertainty of that measurement!* **Always!**

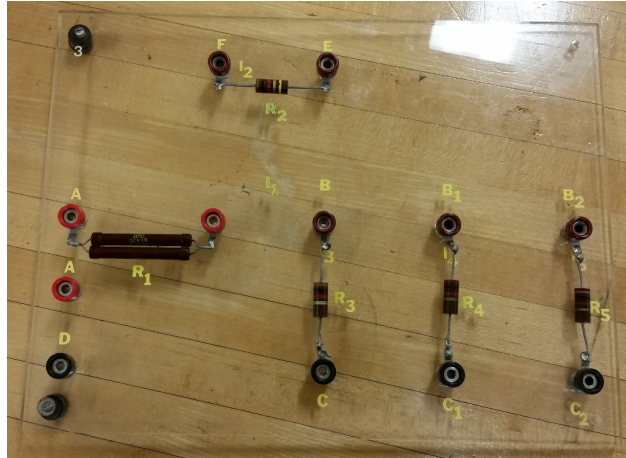


Figure 3: The circuit board used in this lab. You only need to use three of the resistors on this board; while you can use any three just as easily, we strongly suggest using the three parallel resistors at the bottom right.

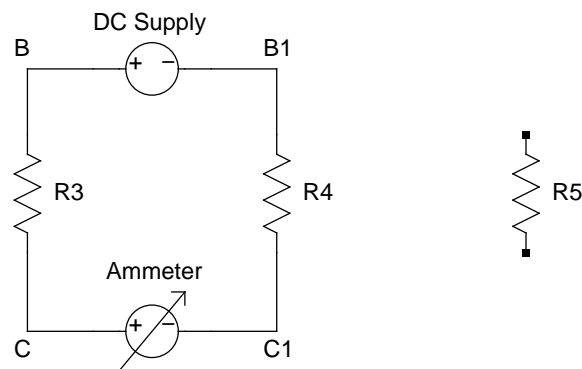


Figure 4: The circuit used in Section 4.1 to study Kirchoff's Loop Rule.

Remember also that the *direction* of your measurement matters: if you swap the measurement leads, you change the *sign* of the measured current or voltage (why?). The same holds for power supplies. You must come up with a measurement convention, and consistently apply it, or you will get the wrong results. The circuit board has five resistors on it, but you will only use three of them for this lab; you can choose any three, but it is probably easiest to use the three parallel resistors in the bottom right corner of the board; see Figure 3.

## 4.1 One loop

You will use the circuit in Figure 4 to study the Loop rule; note that you will not use all the resistors on the board. Do *not* build this circuit yet!.

1. Before connecting any components, make sure to turn the power supply voltage *on*

*both power supplies* down to zero!

2. Measure and record the resistance of each resistor on the board. Do these measurements match the color code on each resistor? They may not, but if they are in the ball park, you may use them anyway.
3. Calculate the maximum safe current for the circuit; determine which ammeter inputs you need to use so as not to blow the fuses in the ammeter! When in doubt, start with the largest range available!
4. Connect the components as shown in the schematic, and slowly turn up the voltage to a value that will not exceed the maximum safe current in any of the resistors. Something like 15 V should be a reasonable starting value. **Important!:** If the ammeter reading “jumps” from zero to a very high (or offscale) value when you start turning up the voltage, disconnect the circuit and get help from your Instructor . . . do this *before* you smell smoke or blow the fuses. Also, if the power supply “refuses” to produce a voltage high enough for your liking, get help from your instructor to adjust the current limiter in the power supply.
5. Measure and record the current in the loop with the ammeter. Remember that currents are *directed*; this means you need to measure with the probe leads oriented consistently.
6. Measure and record the voltage drops around the loop: pick a junction as you “reference point”, and march around the loop measuring the voltage drops across the power supply, the resistors and the ammeter. Remember that the sum you will do is *directed*; you should use the same orientation convention you used to measure the current. You should find that the voltage drop across the power supply has opposite sign to the voltage drops across all the other components.
7. Repeat the measurements at another voltage to check for consistency.

You should at this point quickly check your data for consistency with Kirchoff’s Loop Rule. Before moving on, reduce the power supply voltage back to zero.

## 4.2 Two loops

For this section, you will use the circuit in Figure 5; note that you will *still* not use all of the resistors on the board.

1. Deja Vu: Before connecting any components, make sure to turn the power supply voltage *on both power supplies* down to zero!
2. Make sure that you in fact measured and recorded the resistances of each and every resistor on the board.
3. Connect the components as show in the schematic. Be sure to match the polarities of both the power supplies and the ammeters.

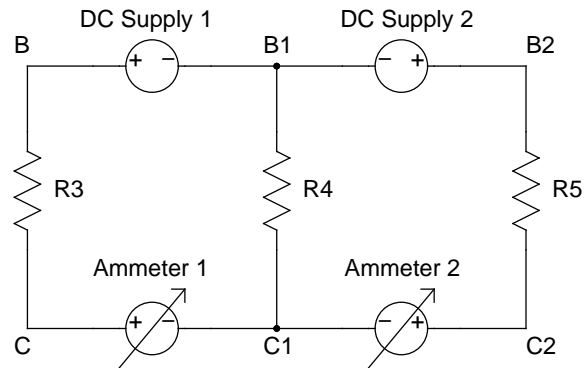


Figure 5: The circuit used in Section 4.2 to study Kirchoff's Loop and Junction Rules.

4. Now, slowly turn up both power supplies to something like 15 V. Again, watch for indications that you have made a wiring error, and turn things off *before* you break things, and *before* getting help from your instructor.
5. Measure and record the currents in both ammeters. You'll also need the current in the shared branch (through resistor  $R_4$ ); you'll get the data for this in the next step.
6. Measure and record the voltage drops across both power supplies, all three resistors, and both ammeters.
7. Repeat the measurements two more times, with a different value of the voltage on one or the other of the power supplies.

At this point, you should quickly check your data for consistency with Kirchoff's Loop and Junction Rules. You get the current through resistor  $R_4$  from Ohm's Law; watch the signs!

## Pre-Lab Exercises

Answer these questions as instructed on Blackboard; make sure to submit them before your lab session!

1. Kirchoff's Rules are defined in terms of loops and junctions. How would you define a "loop" in a circuit? What is a junction?
2. What happens to the reading of the ammeter if you reverse the connections? The voltmeter?
3. Recall that current is just the motion of electric charges. The Junction rule is a practical manifestation of what more fundamental conservation law? What conservation law implies the loop rule?
4. Write a junction equation for the point  $B_1$  of Figure 5. Make clear your conventions for currents
5. Write a loop equation for the loop  $BB_1C_1CB$  of Figure 5.

## Post-Lab Exercises

1. For the one-loop circuit of Section 4.1, verify the Loop Rule. Is the rule satisfied? To what level of uncertainty?
2. For the two-loop circuit of Section 4.2, verify both the Loop and Junction Rules. There are two coupled loops, so you have three total loops to analyze:  $BB_1C_1CB$ ,  $B_1B_2C_2C_1B_1$ , and  $BB_1B_2C_2C_1CB$ . Are the rules satisfied in all cases? To what level of uncertainty?
3. Discuss briefly whether you have met the objectives of the lab exercises.