Experiment 8. Moment of Inertia of a Flywheel

Flywheel is a device for storing large quantities of energy. Can you think of its practical applications?

Objective:
To determine the moment of inertia of a flywheel.

Apparatus:
A flywheel, a timer, a meter stick, a hanger and weights.

Theory:
According to Newton's law, \( F = Ma \), where \( F \) is the resultant of the external forces acting on the body, \( 'a' \) is the linear acceleration of the body and \( M \) is its mass. The analogous relation for rotational acceleration is

\[
\Sigma \tau = I \alpha. \tag{1}
\]

Here \( \Sigma \tau \) is the resultant of external torques acting on the body about the axis of rotation, \( \alpha \) is the angular acceleration and \( I \) is the moment of inertia of the body about the axis of rotation. The kinetic energy of a mass \( M \) having a linear velocity \( v \) is given by

\[
K = \frac{1}{2} mv^2. \tag{2}
\]

In an analogous manner, the kinetic energy of a body of moment of inertia \( I \) and having an angular acceleration \( \omega \) is given by

\[
K = \frac{1}{2} I \omega^2. \tag{3}
\]

Thus, in rotational motion, the moment of inertia plays a role which is analogous to the role of mass \( M \) in linear motion. The cgs unit of moment of inertia is gm.cm\(^2\). The moment of inertia of a body depends on the axis of rotation and the distribution of mass about the axis of rotation.

Equation (3) indicates that a rotating body having a large moment of inertia, like a flywheel, can be used to store large amounts of kinetic energy.

Let a mass \( m \) be attached to the free end of a string wound around the axle of a flywheel as shown in Fig. 1. Further, let \( r \) be the radius of the axle and \( T \), the tension in the string. If the linear acceleration of mass \( m \) is 'a' downward, then by Newton's second law of motion,
\[ T - mg = - ma, \]

or \[ T = m(g - a). \] (4)

The torque acting on the flywheel due to tension \( T \) in the string is given by

\[ \tau = rT \] (5)

Now if \( \tau' \) is the torque due to the frictional forces acting on the flywheel and if \( \alpha \) is the angular acceleration of the flywheel, then Eq. (1) yields

\[ \tau - \tau' = I \alpha \] (6)

Fig. 1

Flywheel

Axle

T

m

\[ T \]

mg

Fig. 2

Flywheel

Axle

Axis of rotation

The linear acceleration \( 'a' \) can be determined by measuring the time taken by the mass \( m \) to fall from rest through a distance \( d \). In such case,

\[ d = \frac{1}{2} at^2, \] because the initial velocity is zero.

Thus \[ a = \frac{2d}{t^2}. \] (7)

The torque \( \tau \) can be determined by using Eqs. (4) and (5), and \( \alpha \) can be calculated by \( a = r \alpha \).

By determining a number of pairs of values of \( \tau \) and \( \alpha \) (for different values of \( m \)), and by plotting a graph between \( \tau \) and \( \alpha \), we shall get a straight line graph according to Eq. (6). Here \( \tau' \) is assumed to be constant. If \( (\tau_1, \alpha_1) \) and \( (\tau_2, \alpha_2) \) are the coordinates of two points on this graph, then
\( \tau_1 - \tau' = I \alpha_1 \) and \( \tau_2 - \tau' = I \alpha_2 \).

By subtracting, we get \( \tau_2 - \tau_1 = I(\alpha_2 - \alpha_1) \).

Or \( I = \frac{\tau_2 - \tau_1}{\alpha_2 - \alpha_1} \). \hfill (8)

If the flywheel is a circular disk of mass \( M \) and radius \( R_1 \), the theoretical value of its moment of inertia is given by

\[
I = \frac{1}{2} MR_1^2. \hfill (9)
\]

The radius of gyration (\( k \)) of a body of moment of inertia \( I \) and mass \( M \) is defined by the relation \( I = Mk^2 \).

Thus \( k = \sqrt{\frac{I}{M}} \). \hfill (10)

A particle of mass \( M \) placed at a distance \( k \) from the axis of rotation will have the same moment of inertia as that of the flywheel.

Procedure:

1. Determine \( d \), the distance of fall of mass \( m \) by measuring the length of the string (including the height of the hanger). Record the mass of the hanger.

2. Place a suitable mass on the hanger, wind the string around the axle (the black disk attached to the side of the flywheel) and place the hanger on the small circular platform under the flywheel. Trip the platform and simultaneously start the timer. Stop the timer as soon as the string gets detached from the small peg on the axle.

3. Repeat step 2 by changing the mass on the hanger 4 or 5 times.

4. Measure the diameter of the axle. Record the radius and mass of the flywheel.
1. In an experiment, a mass $m = 40 \text{ gm}$, attached to a string wrapped around the axle of a flywheel and starting from rest, falls through a distance of 147 cm in 3.5 s. The diameter of the axle of the flywheel is 12 cm. What is the acceleration of the mass and the angular acceleration of the flywheel?

2. Find the tension in the string.

3. Find the torque due to the tension acting on the flywheel.

4. Assuming that the torque due to friction is negligible, find the moment of inertia of the flywheel.
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<tr>
<td>Partner</td>
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<td>Objective:</td>
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<td>Theory/Formulas:</td>
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Exp 8-6D

Data Sheet

Readings of the diameter of the axle:
Reading 1 =
Reading 2 =
Average diameter of the axle =
Radius of the axle, r =

Readings of the distance of fall:
Reading 1 =
Reading 2 =
Average distance of fall d =
Mass of the flywheel, M₁ =
Radius of the flywheel, r₁ =

Readings of time of fall:

<table>
<thead>
<tr>
<th>Reading No.</th>
<th>Falling mass m (including the mass of the hanger)</th>
<th>Time of fall</th>
<th>Time t₁</th>
<th>Time t₂</th>
<th>Time t₃</th>
<th>Average (t)</th>
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Calculations:

<table>
<thead>
<tr>
<th>Reading No.</th>
<th>Mass m</th>
<th>Average t</th>
<th>Linear accln. ‘a’</th>
<th>Angular accln. α</th>
<th>Tension T</th>
<th>Torque τ</th>
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Plot a graph between τ and α.
Find the moment of inertia from the slope of the graph.

\[ \tau_1 = \quad \alpha_1 = \]

\[ \tau_2 = \quad \alpha_2 = \]

Experimental value of moment of inertia,

\[ I_{\text{exp}} = \]

Theoretical value of moment of inertia,

\[ I_{\text{th}} = \]

Percent error =

Radius of gyration of the flywheel,

\[ k = \]
Experiment No. 8: Questions

1. What is meant by moment of inertia?

2. Name the forces acting on the descending mass.

3. Name the torques acting on the flywheel when the mass is descending. Do these torques remain constant during the fall of the mass?

4. What happens to the potential energy lost by the falling mass?

5. Describe a practical application of flywheel.