The Electrical Properties of Materials: Resistivity

1 Objectives

1. To understand the properties of resistance and resistivity in conductors,
2. To measure the resistivity and temperature coefficient of tungsten, and
3. To design and implement a resistivity measurement experiment.

2 Introduction

In previous lab(s) you have measured the resistance of different objects via different methods. You have also used the known resistances of objects to make predictions about voltages across and currents through those objects. After a few moments thought, however, it should be clear that the resistance of an object is not a fundamental property of the materials the object is constructed from: it is a derived property that depends not only on the material, but also on the geometry of the material. The analogy of current flow to fluid flow is apt here: recall from our study of fluid dynamics last semester that water has an “easier” time flowing through a short or large diameter pipes than through long or small diameter pipes.

The resistance of an object, then, is not a fundamental property of the material: it is derived, or extrinsic. From studying how resistance varies with shape (geometry), you can determine that there is a property of the material that is fundamental, or intrinsic, to the material. We call this property resistivity.

While we consider resistivity an intrinsic property of the material, that does not mean it is a constant! Under different temperature conditions, for instance, the resistivity changes. In this lab, you will explore the properties of resistivity of various materials.

3 Theory

The resistance of a real (as opposed to an ideal) conductor of uniform cross section is given in terms of the resistivity, $\rho$, length, $L$, and cross-sectional area, $A$, by the relation

$$R = \rho \frac{L}{A}.$$  

1The inverse of resistivity is also given a name, the conductivity. In some older physics texts and in more engineering oriented texts, you may see these referred to as “specific resistance” and “specific conductance”, in analogy with the thermodynamic properties of “heat capacity” and “specific heat”.
This is a single relation between four variables; if you know any three of them, you can calculate the fourth. Dimensional analysis should convince you that the SI units of resistivity are $\Omega \cdot \text{m}$.

The resistivity of a material depends on the temperature. In general, much as the length of a material increases with temperature, so does the resistivity. Detailed models of the temperature dependence of resistivity are very complicated. However, if you look over a narrow temperature range, you can model the temperature dependence with a linear function, just as you did for the temperature dependence of length. So, if $\rho_1$ and $\rho_2$ are the resistivities at temperatures $T_1$ and $T_2$ respectively, you can say

$$\rho_2 = \rho_1 \left(1 + \alpha(T_2 - T_1)\right),$$

where $\alpha$ is an empirically (experimentally) determined parameter called the temperature coefficient of resistivity.

4 Procedures

This lab has two parts: a scripted study of the resistivity of tungsten (Section 4.1) and a more free-form determination of the resistivity of various materials. In the latter section, we do not give you detailed instructions: you are responsible for both designing and executing the experiment, and analyzing and presenting your results.

4.1 The Incandescent Bulb

An incandescent bulb consists of a coiled coil of tungsten wire, which has a very small coefficient of thermal expansion. When you apply an electrical current, the filament - which
Figure 2: The variable transformer, aka variac.

is just a resistor, after all - dissipates power according to the relation

\[ P = VI, \]

and heats to a few thousand degrees Celsius. As you recall from Physics I, hot objects emit electromagnetic radiation; the tungsten filaments are designed to emit as large a fraction of their power in the optical spectrum as possible - i.e. as visible light.

Light bulbs are labeled for sale according not to the amount of light they emit, or their resistance, but to their power dissipation (“We need to buy more 60 W light bulbs.”). This rating assumes that the bulb will be operated at a fixed 120 V; what happens to the power dissipation at other operating voltages?

For this set of measurements, you should receive a set of incandescent bulbs (25 W, 40 W, 60 W, 75 W, and 100 W), a variac (variable transformer; see Figure 2), two multimeters, a lamp socket board and connecting wires.

**Warning!**

The light bulbs will get hot! Be careful when you unscrew them so you don’t get burnt. In addition, there will be unprotected 120 V AC sources present. Just as you wouldn’t stick you fingers in a wall socket, do not touch the bare wires or terminals! Don’t stick any part of your hand under the acrylic sheets! Don’t disconnect the wiring from the variac without turning it off!

1. Measure and record the ambient temperature. Why do you need this?

2. Directly measure the resistance of each of your lamps. Using the data in Table 2 you can determine the resistivity of tungsten as predicted by each lamp individually. Are your measurements consistent? If not, what might be the problem?
Table 1: The new circuit schematic symbols for this lab.

<table>
<thead>
<tr>
<th>Variac</th>
<th>Lamp</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Variac Symbol]</td>
<td>![Lamp Symbol]</td>
</tr>
</tbody>
</table>

Figure 3: The circuit including the incandescent lamp powered by the variac.

Table 2: The filament data table.

<table>
<thead>
<tr>
<th>Lamp Wattage [W]</th>
<th>Operating Temperature [°C]</th>
<th>Length of uncoiled filament [m]</th>
<th>Cross-section [m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>2290</td>
<td>0.56</td>
<td>$7.00 \times 10^{-10}$</td>
</tr>
<tr>
<td>40</td>
<td>2470</td>
<td>0.38</td>
<td>$8.56 \times 10^{-10}$</td>
</tr>
<tr>
<td>60</td>
<td>2550</td>
<td>0.53</td>
<td>$1.60 \times 10^{-9}$</td>
</tr>
<tr>
<td>75</td>
<td>2600</td>
<td>0.55</td>
<td>$2.23 \times 10^{-9}$</td>
</tr>
<tr>
<td>100</td>
<td>2650</td>
<td>0.58</td>
<td>$3.17 \times 10^{-9}$</td>
</tr>
</tbody>
</table>
3. Build the circuit shown in Figure 3 which will be run at 120 V AC; we will talk about AC or alternating current at a future date. For this lab, the differences between AC and DC voltages are irrelevant; just follow the directions here. The multimeter measuring voltage should be set to the AC V setting; the multimeter measuring current should be set to the AC I setting, and should use the lower right, red 10 A and right hand, black LO inputs; see Figure 4.

**DO NOT** plug in the variac or turn on the circuit until it has been checked by your instructor!

4. Finally, turn on the circuit, and set the voltage to 120 V by rotating the large knob on the variac. Measure and record both the voltage and current values. Repeat with all of the lamps. You will be determining the temperature dependent resistance of the filaments, and extracting the temperature coefficient of resistivity.

### 4.2 Variations on a theme

In this section, you will be responsible for answering the following question: what is the resistivity of material X? You will have available a high precision multimeter (see Figure 4) with probes, calipers, micrometers, and various bibs and bobs around the lab (like wires, clips, tools, meter sticks, stands, etc.) You must design, implement, and critique an experimental procedure to make a reasonable determination of the resistivity of the current carrying material. If you don’t see something you might find useful, ask; we might just have it available.

Your instructor will split your lab partnerships into groups, which will design and implement the experiment. Discuss, write on the board/paper/tablet, draw, diagram, argue, convince, cajole, and then measure and analyze the results. Don’t be passive: each student must still produce their own writeup, indicating their own ideas and contributions! See the
post-lab exercises for some topics of discussion. Now, go and perform the experiment you just designed!

Here are some of the possible materials to analyze:

1. Pencil lead isn’t; it’s actually a mechanically compressed form of the carbon allotrope graphite. Graphite is not only flammable, it is also useful as a lubricant, and it conducts electricity. Determine the resistivity of the graphite in a standard Number 2 pencil.

2. Most metals are ductile as well as conductive; this is not a coincidence: the most conductive metals (lowest resistivity) also tend to be the most ductile. Wires are drawn from large diameter cylinders, first by rollers, and then by forcing the cylinder through a tapered bore-hole in a heavy-weight die. The result is a wire of uniform (usually circular) cross-section that is wrapped on a spool. Using one of the provided lengths of wire (copper, steel\textsuperscript{2} and nichrome\textsuperscript{3}) determine the resistivity of the material.

3. Sometimes, getting access to the wire whose resistivity you want to measure requires some clever thought. Last semester, you performed a calorimetry experiment, where you heated the insulated volume using an immersed wire coil. Without cutting or

\textsuperscript{2}An alloy of iron and carbon, with traces of other elements.
\textsuperscript{3}A steel-like alloy of nickel, chromium, and iron.
disassembling the calorimeter, determine the resistivity of the coil material. Note: the conductive posts that hold the coil are a different material, and their effect should be eliminated in your measurements.

4. Sometimes, it isn’t the resistivity of the material you’re after: remember, Equation (1) relates four different quantities; given any three, you can determine the remaining one. Given a spool of copper wire of known resistivity (16.8 nΩ · m at 20°C), determine the length of the wire on the spool.
Pre-Lab Exercises

Answer these questions as instructed on Blackboard; make sure to submit them before your lab session!

1. What’s the difference between intrinsic and extrinsic properties of a material?

2. We give a relation between dissipated power in, voltage across, and current through, a resistor. Ohm’s Law gives a relationship between the voltage, current, and resistance. Express dissipated power in terms of the voltage and resistance; in terms of the current and resistance.

3. A perfect conductor has zero resistivity, while a real conductor has a finite, but near zero resistivity. What resistivity should a perfect insulator have? Should a real insulator have a very large or very small resistivity?
Post-Lab Exercises

1. Use the data you collected to determine the resistivity of tungsten; be sure to include an estimate of the measurement uncertainty, and discuss how you determined that uncertainty.

2. Use the data you collected to determine the temperature coefficient of resistivity for tungsten; be sure to include the data points at room temperature, otherwise you are likely to get garbage! Include an estimate of the uncertainty, and discuss how you determined that uncertainty. Since you have the resistivity at six temperatures, you should produce at least one (and better, two!) plots of the resistivity versus temperature; is it linear? Are you sure? Can you extrapolate from the high operating temperatures down to room temperature?

3. By measuring the voltage and current used for each lamp, you can calculate the actual dissipated power. How does this compare to the advertised value? Stefan’s Law (look back at your Physics I text) relates the electromagnetic power dissipation to the surface temperature of an object; since you have both, you can calculate the emissivity of each lamp filament. Are they the same? Speculate on the source of any differences.

4. For the experiment you designed, write a brief “Lab Manual”: state the objectives, write a brief overview of the theory, describe your hypothesis testing procedure. State your results, including any estimate of the uncertainty, and how you reached those estimates. Based on your experience, would you modify your procedure to make the same measurement in the future? If so, how? If not, why not?

5. Discuss briefly whether you have met the objectives of the lab exercises.