

# Content Matters

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# K-12 Mathematics Education is in trouble:

but is the "problem"  
mainly:

*the culture*

*the students*

*the teachers*

*the content*

*the system ?*

# Content matters

but what is best for different students might be different:

- \* content for STEM needs
  - \* content for the needs of all students
  - \* content for the workplace
  - \* content for those "good" at mathematics
  - \* content for those who have "trouble" with mathematics
- etc.

# Goals of all content of mathematics should include:

- \* building ambassadors for mathematics
- \* having future parents who are sophisticated in their views about mathematics and its importance
- \* exposing students to a variety of mathematical tools for solving problems of the workplace and daily life
- \* making students aware of domains where mathematics is applicable
- \* making students aware of careers that use mathematics

# Important considerations:

- \* Mathematical Thinking
- \* Reasoning Skills and Problem Solving
- \* Mathematics Applied
- \* Applicable Mathematics
- \* Technology

Historically we have  
been very  
concerned with  
mathematical  
technique

# Techniques:

0. Arithmetic
1. Geometry
2. Algebra
3. Trigonometry
4. Calculus (Single Variable and Multivariate)
5. Differential Equations
6. Linear (Matrix) Algebra
7. Modern Algebra
8. Probability and Statistics
9. Real Variables
10. Complex Variable
11. Combinatorics
12. Graph Theory
13. Coding Theory
14. Knot Theory
15. Partial Differential Equations

(Many more!)

# Technique:

"older"

- a. Solving linear equations
- b. Solving quadratic equations
- c. Arithmetic of complex numbers

"newer"

- d. Matrix addition and multiplication
- e. Find measures of central tendency
- f. Computing probabilities
- g. Counting permutations and combinations

# Themes:

1. Optimization
2. Growth and Change
3. Information
4. Fairness and Equity
5. Risk
6. Shape and Space
7. Pattern and Symmetry
8. Order and Disorder
9. Reconstruction (from partial information)
10. Conflict and Cooperation
11. Unintuitive behavior

(Many more!)

## *Global Example:*

Cell phones are a gift (curse?) of 20th century mathematics to humanity!

**\* error correction codes and technology**

(seminal work of Richard Hamming)

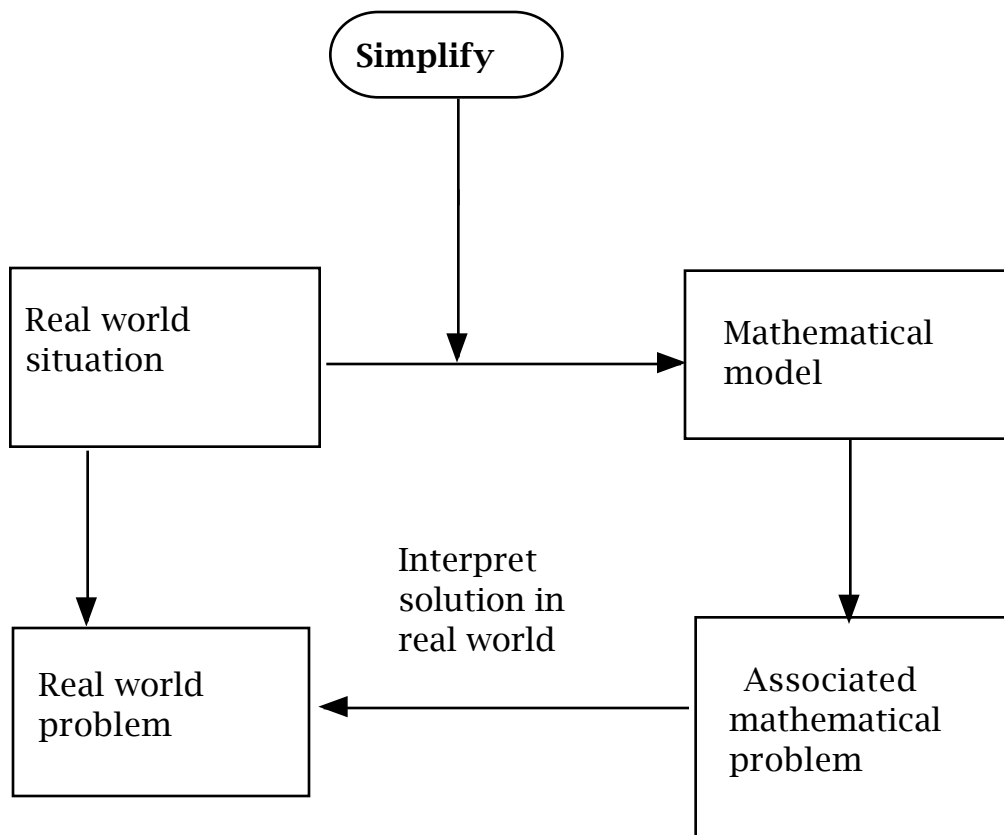
**\* data compression technology**

(seminal work of David Huffman)

**\* global positioning systems**

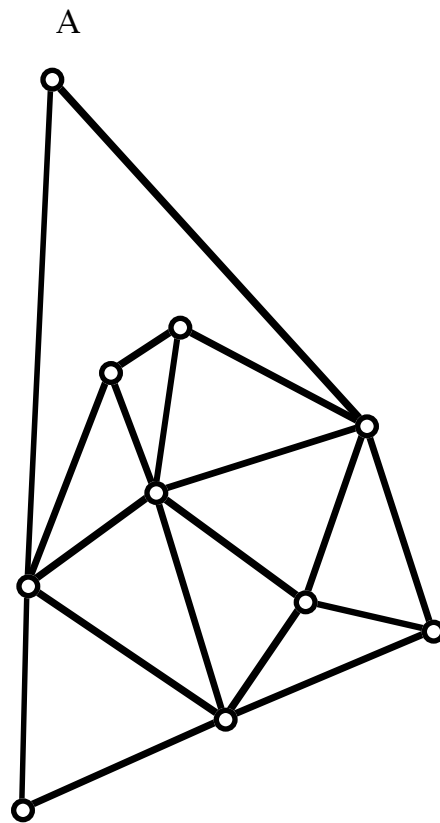
**\* mathematics for frequency assignment of calls**

# We should be teaching themes and techniques in a modeling framework throughout K -12!



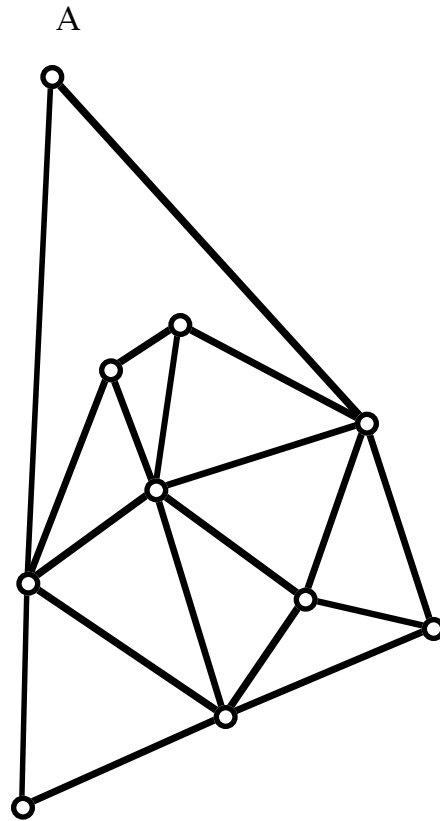
## Example:

Is it possible to find an efficient pot hole inspection route, which starts and ends at A for the street network below?



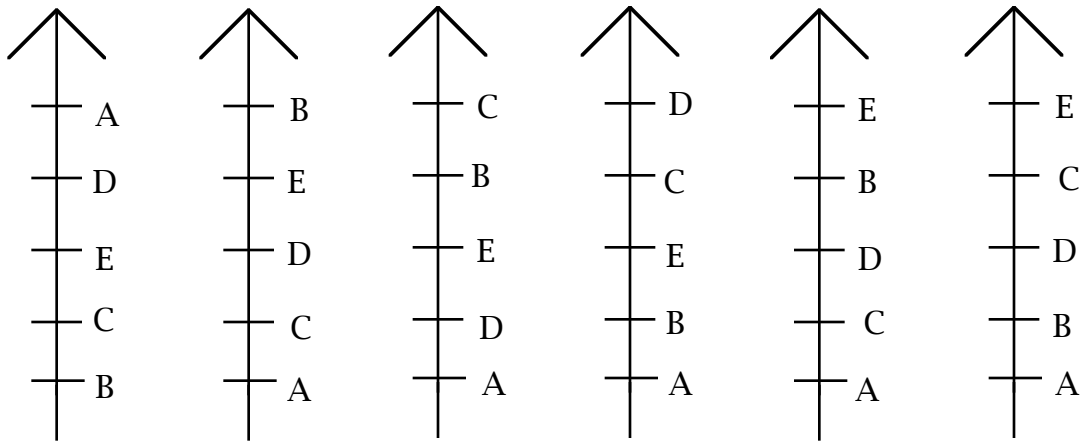
## Example:

Is it possible to find an efficient route, which starts and ends at A and which traverses each of the "bridges" (represented by line segments) in the diagram below below?



# Example:

Who is the most "deserving" winner of this election and why?



Votes: 18

12

10

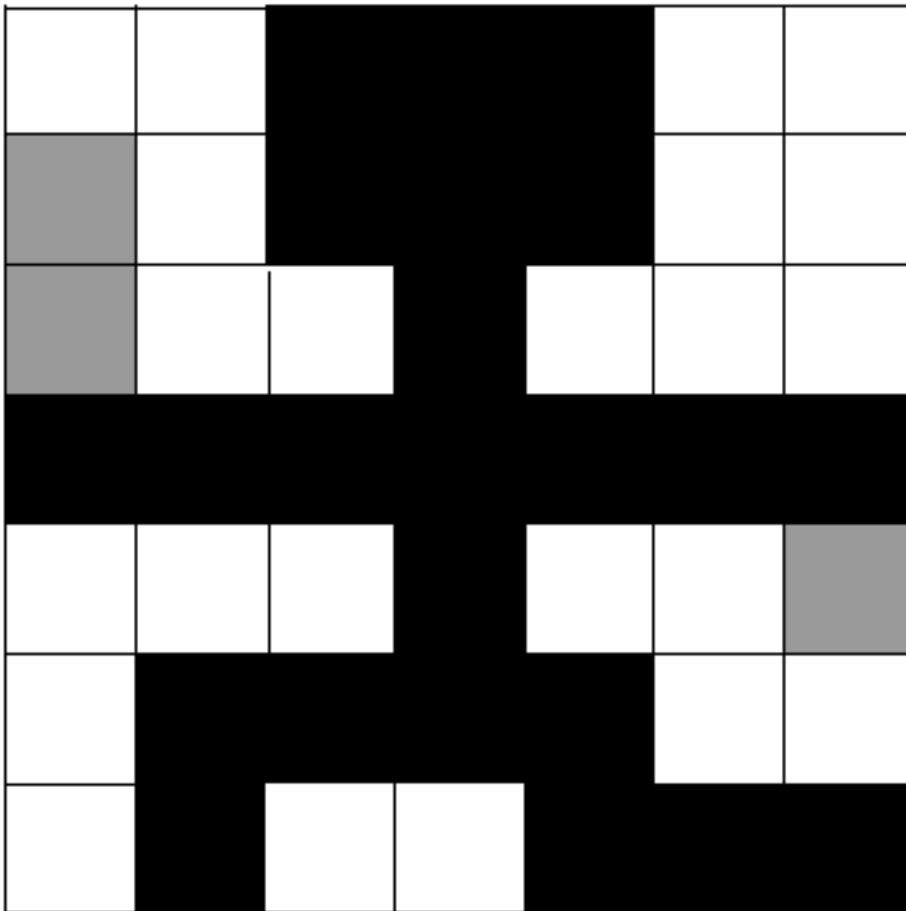
9

4

2

## Example:

How would you send this image over a phone connection?



# Example:

## Is this a fair game?

		Column	
		C <sub>1</sub>	C <sub>2</sub>
Row	R <sub>1</sub>	5	-2
	R <sub>2</sub>	-10	4

Spell checker?

How does a spell checker work?

Idea:

Distance between two strings of the same length is the number of positions they differ in:

100011

001101

*Hamming distance* is 4

If you enter **bneak** into a word processor,

*bneak* is not a word in its dictionary

The word processor suggests words which are Hamming distance 1 from **bneak** under the assumption that you made one "typo."

*bleak*

*break*

*sneak*

Ideas like this are what made it possible to find distances between DNA strands and the songs of different birds, and the faces of different people!

*Example:*

A bankrupt company with assets of \$180 million must pay off two claimants with verifiable claims of:

A = \$300 million

B = \$200 million

What is a fair way for a Judge to settle the claims?

*What fairness principle should one use?*

\* equality of what the players are given?

\* equality of what the players lose?

\* proportionality to claims?

\* return fixed percentages on each claimed dollar?

etc.

Methods of solving such problems lead to many interesting methods and ideas:

- \* Maimonides method
- \* Contested garment rule
- \* Constrained optimization
- \* Shapely value

# Principle:

Equalize loss:

If A given a

and B given b, then:

$$a + b = 180$$

$$300 - a = 200 - b$$

So solve:

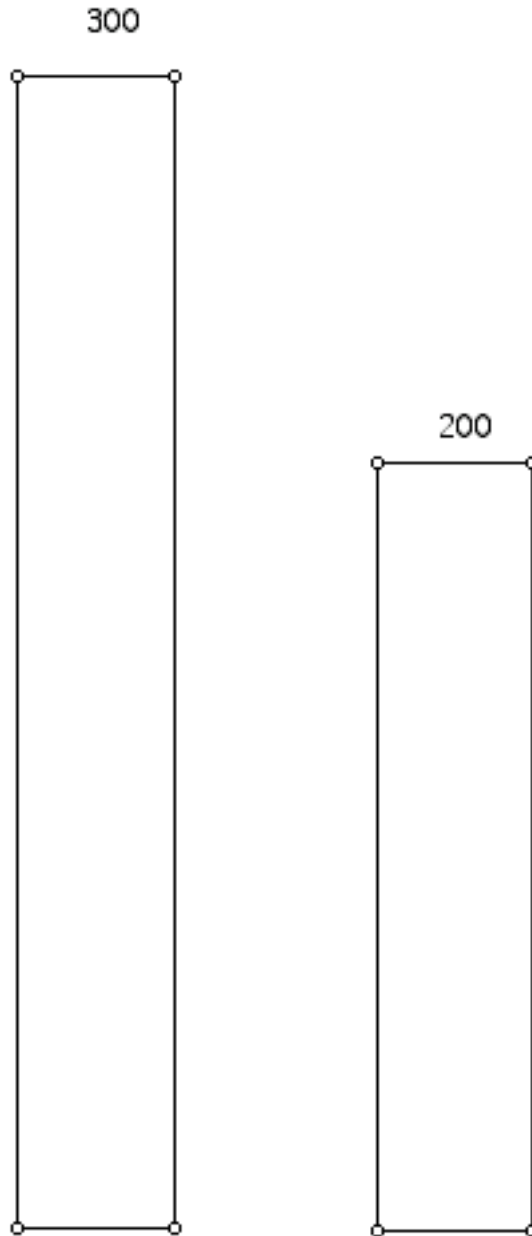
$$a + b = 180$$

$$-a + b = -100$$

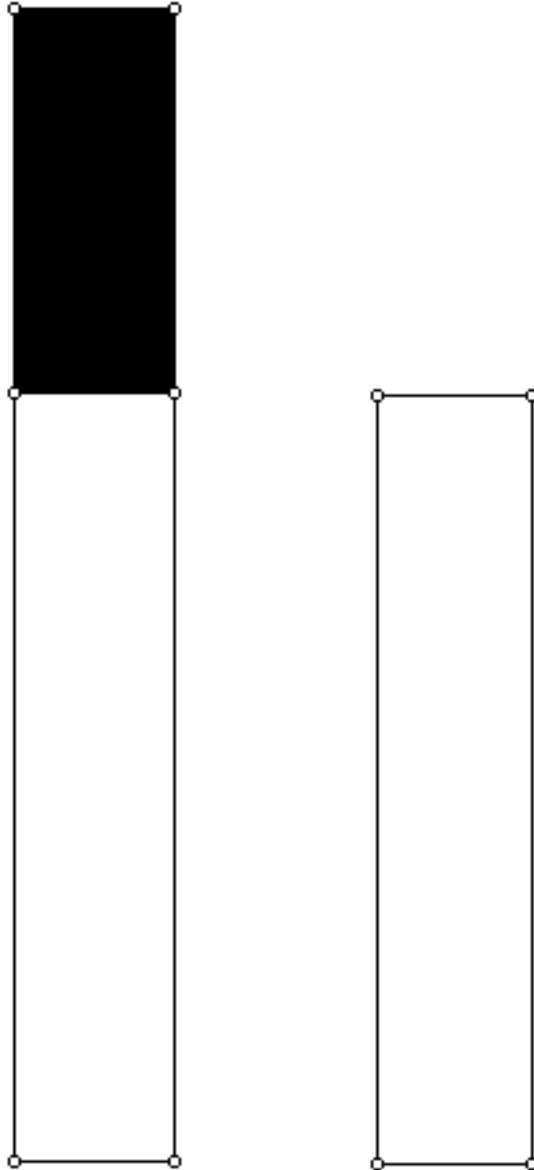
$b = 40$  and, hence,  $a = 140$ .

# Geometric Solution:

Diagram showing the losses for A and B before any money is returned:

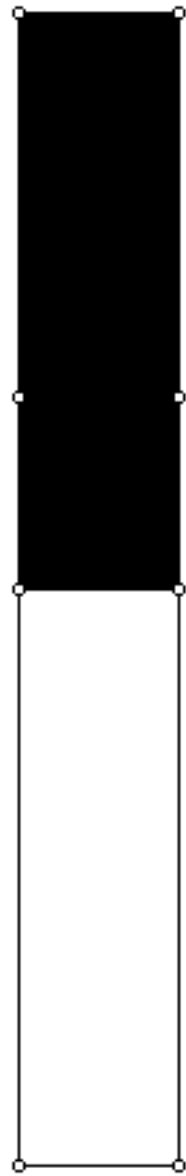


We must first give A enough back so that the loss for A and B are the same!



Now A and B have equal losses but there is only 80 left to distribute. So we give 40 units to each of A and B:

140 units to A



300

A's loss is 160

40 units to B



200

B's loss is 160

We do not stop at  
1900 in teaching  
history in our  
schools because  
there is so much  
important history  
prior to 1900!

We should not stop  
teaching  
mathematics  
developed after  
1900 because there  
is so much  
important  
mathematics  
developed prior to  
1900!

# Content for teachers:

- \* high school mathematics teachers should major in mathematics

- \* you can not teach about topics you know nothing about

- \* teachers must know mathematics beyond what they actually teach

Content matters!

Students must  
develop reasoning  
skills, algorithmic  
skills, and  
modeling skills  
throughout K -12!